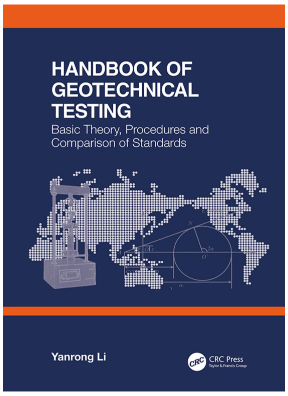

Handbook of Geotechnical Testing
Basic Theory, Procedures and Comparison of Standards

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Characteristics of rocks

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Chapter 2

Characteristics of rocks

In civil engineering, rocks refer to geological materials that cannot be crushed by hand squeezing or be partially scrunched. In geology, rocks pertain to lithified solid materials of igneous, sedimentary, pyroclastic or metamorphic origin.[1] The physical properties of rocks include porosity, density, water absorption and hydraulic properties. Among them, rock porosity exerts important influence on the mechanical properties of rocks. Void spaces in rocks include pores and micro-fissures. Pores affect water flow in rocks. Conversely, the micro-fissures affect the distribution of the deformation and internal stress of rocks. The properties of micro-fissures are as important as the minerals in the rock itself. Micro-fissures can reduce rock strength. Whether the micro-fissures close or not can lead to discrete differences in the rock test results. Griffith's strength theory explains the effects of micro-fissures on tensile strength and the brittle failure of rocks. Commonly used rock failure theories include Mohr strength theory, which mainly deals with the stress analysis of rock shear failure.

2.1 Physical properties

Rocks consist of three phases: solid, liquid and gas. The specific gravity of each phase in a rock has an important influence on the physical properties of the rock. The physical properties of rocks mainly include porosity, density and hydraulic properties, and such features can reflect the strength and weather resistance of rocks.

2.1.1 Porosity

In rocks, the term 'voids' is generally used to refer to pores and fissures. Voids are classified as closed and open according to their compatibility with the outside world. Open voids include large and small open voids. At normal temperature, water can enter large open voids but not small open voids. Only in a vacuum or under 150 atmospheric pressure can water enter small open voids.

Indexes that reflect rock porosity include total void ratio, large open porosity, small open porosity, total open porosity, closed void ratio and void ratio.[2] The commonly mentioned 'rock void' pertains to the total void ratio of the rock.

(1) Total void ratio (n): The ratio (in percentage) of the volume of the void (V_v) to the total volume (V) of the rock specimen. It is calculated as

$$n = \frac{V_v}{V} \times 100\%. \tag{2.1}$$

(2) Large open porosity (n_b): The ratio (in percentage) of the volume of the large open void (V_{vb}) to the total volume of the test specimen (V). It is calculated as

$$n_{nb} = \frac{V_{vb}}{V} \times 100\%. \tag{2.2}$$

(3) Small open void ratio (n_a): The ratio (in percentage) of the volume of the small open void in the rock specimen (V_{va}) to the total volume (V) of the test piece. It is calculated as

$$n_l = \frac{V_{va}}{V} \times 100\%. \tag{2.3}$$

(4) Total open void ratio (n_0): The ratio (in percentage) of the volume of the open void in the rock specimen (V_{v0}) to the total volume (V) of the test piece. It is calculated as

$$n_0 = \frac{V_{v0}}{V} \times 100\% \tag{2.4}$$

(5) Closed void ratio (n_c): The ratio (in percentage) of the volume of the closed void in the rock specimen (V_{vc}) to the total volume (V) of the test piece. It is calculated as

$$\mathrm{n}_c = \frac{V_{vc}}{V} \times 100\%. \tag{2.5}$$

(6) Void ratio (e): The ratio of the volume of the void (V_v) to the volume of the solid mineral particles (V_s) in the rock specimen. It is calculated as

$$\mathrm{e} = \frac{V_v}{V_s} = \frac{V - V_s}{V_s} = \frac{n}{1-n}. \tag{2.6}$$

Sedimentary rocks are formed by the accumulation of mineral particles, rock fragments or shells. Porosity varies from 0 to 90%, and the typical value of sandstone porosity is 15%. In sedimentary rocks, the void fraction generally decreases as the age of deposition increases. Statistically, chalk is the most porous of all rocks, and in some cases, its porosity can exceed 50%. Some lava (such as pumice) can also exhibit extremely high void rates because of the preservation of volcanic bubbles. The void ratio is usually less than 1% or 2% in fresh igneous rocks, and as the degree of weathering increases, the void ratio can increase to more than 20%.[3]

2.1.2 Density

The densities of rock indexes include grain density, natural bulk density, dry bulk density and saturated bulk density. A rock density index can be measured by a grain density test and a bulk density test. Full details about the grain density and bulk density tests are given in GB/T 50266-2013 and ASTM D4644-16.

(1) Grain density (ρ_s)
The grain density of rocks, also called true density, is the ratio (in g/cm[3]) of the rock mass m_s to the solid volume V_s. Rock grain density can be calculated as

$$\rho_s = \frac{m_s}{V_s}. \tag{2.7}$$

(2) Natural bulk density (ρ)
The natural bulk density of a rock refers to the mass of a rock per unit volume of the rock in a natural state, that is, the ratio (in g/cm[3]) of mass m to volume V. The natural bulk density of a rock can be expressed as

$$\rho = \frac{m}{V}. \tag{2.8}$$

The natural bulk density of a rock depends on its mineral composition, pore development and water content. This index reflects the merits of rock mechanics to some extent. In general, the greater the natural bulk density, the better the mechanical properties, and vice versa.

(3) Dry bulk density (ρ_d)
The dry bulk density of a rock is the ratio (in g/cm[3]) of the solid mass m_s in the rock to the volume V of the rock mass. It is calculated as

$$\rho_d = \frac{m_s}{V}. \tag{2.9}$$

(4) Saturated bulk density of a rock (ρ_{sat})
The saturated bulk density of a rock refers to the ratio (in g/cm[3]) of the mass of the rock when the pores are filled with water to the volume of the rock. It is calculated as

$$\rho_{sat} = \frac{m_s + V_v \times \rho_w}{V}. \tag{2.10}$$

The density parameters of rocks are important for engineering practice. In general, rock density is positively correlated with rock strength and elasticity. For oil shale deposits, density can also indicate the value of mineral commodities. In the coal industry, a strong correlation exists between density and ash content and pre-cover depth.

2.1.3 Hydraulic properties

The hydraulic properties of rocks refer to their features underwater or under water immersion conditions. These properties include water absorption, swelling, slake-durability, softness and frost resistance.

(1) Water content and water absorption

The ratio (in percentage) of the mass m_w of the water to the mass m_s of the rock in the natural state is called the water content (water content) ω. It is calculated as

$$\omega = \frac{m_w}{m_s} \times 100\%. \tag{2.11}$$

Full details about the test are given in GB/T 50266-2013 and ASTM D2216-10. Water content is a crucial parameter for weakening rocks. Weak rocks usually contain a large quantity of clay minerals that are easily softened by water. Water content has an important dominating effect on rock strength and deformation.[4]

The capability of rocks to absorb water under certain conditions is called its water absorption capability. Commonly used water absorption indexes include water absorption, saturation rate and coefficient of saturation. The test parameters for the rock water absorption index are given in GB/T 50266-2013 and ASTM D6473-15.

1) Water absorption rate (ω_a)
   The water absorption rate of rock refers to the ratio (in percentage) of the mass m_{w1} of water absorbed by rocks under normal atmospheric pressure and room temperature to the mass of rock solids, and is calculated as

$$\omega_a = \frac{m_{w1}}{m_s} \times 100\%. \tag{2.12}$$

2) Saturated water absorption rate (ω_{sa})
   Saturated water absorption refers to the ratio (in percentage) of the mass m_{w2} of the water absorbed by the rock under high pressure or vacuum to the solid mass of the rock. It is calculated as

$$\omega_{sa} = \frac{m_{w2}}{m_s} \times 100\%. \tag{2.13}$$

3) Coefficient of saturation (η_w)
   The coefficient of saturation is the ratio of the water absorption rate to the saturated water absorption rate and is calculated as

$$\eta_w = \frac{\omega_a}{\omega_{sa}}. \tag{2.14}$$

The water absorption of rocks is closely related to the pores inside the rocks. The water absorption rate of a rock can reflects the development degree of its large open pores. The greater the water absorption rate, the larger the ratio of the large open pores in the rock. The saturated water absorption of a rock indicates the development degree of its open pores; the larger the saturated water absorption, the greater the number of open voids. The coefficient of saturation represents the relative proportional relationship between the open and closed pores in a rock.

The larger the saturation coefficient, the greater the large open pore proportion in the rock, and the lower the number of remaining pores after water absorption at normal temperature and pressure. This kind of rock is susceptible to frost heaving damage because of poor frost resistance.

(2) Swelling

The phenomenon in which the rock volume expands after water absorption and causes structural damage is called rock swelling. Rock swelling is generally caused by clay minerals in the rock (especially montmorillonite). The indexes for evaluating swelling include the axial free swelling ratio (V_H), radial free swelling ratio (V_D), lateral constrained swelling ratio (V_{HP}) and swell pressure (p_e).

Axial free swelling: The rock expands freely in water, and the ratio of axial deformation ΔH to the original height H of the sample is called axial free swelling ratio which is calculated as

$$V_H = \frac{\Delta H}{H} \times 100\%. \tag{2.15}$$

Radial free swelling: The rock expands freely in water, and the ratio of radial deformation ΔD to the sample diameter or side length D is called the radial free swelling ratio which can be calculated as

$$V_D = \frac{\Delta D}{D} \times 100\%. \tag{2.16}$$

Lateral constrained swelling: The rock under lateral restraint expands in water, and the ratio of the axial deformation ΔH_1 to the original height H of the sample is called the lateral constrained swelling ratio which is calculated as

$$V_{HP} = \frac{\Delta H_1}{H} \times 100\%. \tag{2.17}$$

Swell pressure is the maximum pressure applied to maintain the volume of the rock as it expands in water. Full details about the test for rock swelling index are given in GB/T 50266-2013.

(3) Slaking

The enduring slaking index I_{d2} is the main slaking index of rocks. It is measured by the dry–wet cycle test. The said index is the ratio of the dry mass m_r of the sample after two dry–wet cycles to the dry mass m_s of the sample before the test.

$$I_{d2} = \frac{m_r}{m_s} \times 100\%. \tag{2.18}$$

Full details about the test are given in GB/T 50266-2013 and ASTM D4644-16.

(4) Softness

Strength reduction after rock immersion is called the softening of the rock and is measured by the softening coefficient η_c. The softening coefficient refers to the ratio of the compressive strength σ_{cw} of the rock sample in the saturated state to the compressive strength σ_c in the dry state. It is calculated as

$$\eta_c = \frac{\sigma_{cw}}{\sigma_c}. \tag{2.19}$$

Rock softness is related to the hydrophilicity of rocks, their soluble mineral contents and open pores. The coefficient of softness is one of the important indexes for evaluating the mechanical properties of rocks. It is mostly used in hydraulic construction to evaluate the stability of a dam foundation rock mass. That is, when $\eta_c > 0.75$, rock softness is weak, and the engineering geological properties are satisfactory; conversely, when $\eta_c < 0.75$, rock softness is strong, and the engineering geological properties are poor.

(5) Frost resistance

The ability of rocks to resist freeze–thaw damage is called frost resistance, a quality expressed by the coefficient of frost resistance R_d and mass loss rate K_m. The rock frost resistance index can be measured by a rock freeze–thaw test. Full details of the operation are given in GB/T 50266-2013 and ASTM D5312/D5312M-12. The coefficient of frost resistance is the ratio (in percentage) of the dry compressive strength σ_{c2} of the rock specimen after repeated freezing and thawing to the dry compressive strength σ_{c1} before freezing and thawing; it is calculated as

$$R_d = \frac{\sigma_{c2}}{\sigma_{c1}} \times 100\%. \tag{2.20}$$

The mass loss rate refers to the difference between the dry mass before and after freezing and thawing $(m_{s1} - m_{s2})$ and the dry mass m_{s1} before the test, as expressed in percentage and calculated as

$$K_m = \frac{m_{s1} - m_{s2}}{m_{s1}} \times 100\%. \tag{2.21}$$

The main factors that affect rock frost resistance include the thermo-physical properties and strength of rock minerals, the intergranular connections, the development of open pores and water content.

2.1.4 Rock description

The description of rocks is primarily used to provide basic information on their properties. A complete rock description should include strength, color, structure, name, particle size, weathering state and other geological characteristics.[2]

(1) Strength

Rock strength can be described as 'hard' or 'weak', and it has a certain relationship with the point load strength and uniaxial compressive strength index (Table 2.1). The relationship between the uniaxial compressive strength σ_c and the point load strength index $I_s(50)$ is $\sigma_c = 25I_s(50)$.

(2) Color

Rock color is usually presented using three basic parameters: hue, purity and brightness. Hue refers to the basic colors or the mixture of basic colors. Purity pertains to the degree of vividness of color, and brightness denotes the degree of lightness and darkness of color. The description should indicate whether wetting the rock sample reduces its brightness (making the sample darker) but does not change its hue and purity. Therefore, the degree of wetting of the sample should be stated when describing rock color.

(3) Structure and fabric

'Structure' is a broad term that usually refers to the general physical appearance of rocks. It encompasses the size and shape of the particles or crystals (geometrically), the distribution of particle sizes and the degree of crystal development (the relationship between geometric features). This term is mostly applicable to scale features visible to the naked eye in specimens. Rocks that consist of very fine particles must be observed with a microscope.

'Fabric' refers primarily to the arrangement of particles or crystals in a rock. For magmatic rocks and other crystalline rocks, organisation refers to the shape and arrangement direction of the crystalline and amorphous bodies of the rocks. For sedimentary rocks, the fabric focuses on the orientation of individual particles and the location of the particles relative to the cementitious material. In addition, the fabric also includes small structural faces or planes (often referred to as micro-cracks) that pass through the particles or between crystals. The description of microfractures should include their strength, spacing, continuity and direction.

Table 2.1 Classification of rock strengths

Descriptive term	*Uniaxial compressive strength (MPa)*	*Approximate point load strength index values ($I_{s(50)}$) (MPa)*
Extremely weak	<0.5	Generally, not applicable
Very weak	0.5–1.25	
Weak	1.25–5	
Moderately weak	5–12.5	0.2–0.5
Moderately strong	12.5–50	0.5–2
Strong	50–100	2–4
Very strong	100–200	4–8
Extremely strong	>200	>8

(4) Weathering and alteration

Weathering can be divided into chemical weathering and physical decomposition, and these two types usually exist simultaneously. Chemical weathering refers to weathering caused by chemical reactions such as hydration, oxidation, ion exchange and dissolution. The chemical weathering of rocks changes their surface color. Therefore, the degree of chemical weathering can be preliminarily judged by the changes in rock color. The degree of chemical weathering of rocks is divided into six levels, from fresh rock to residual soil. Physical decomposition mainly denotes the changes in the surface stress state caused by the thermal expansion and contraction of rocks. Such change leads to rock rupture. The decomposition of rock materials can also be caused or hastened by biological action (such as hydro splitting).

Alteration pertains to changes in properties caused by the circulation of invading hot gases and fluids. Common terms used to describe alteration are kaolinization and mineralisation.

2.2 Deformability of rocks

The constitutive relations of rocks are primarily reflected in the complete stress–strain curve. Rock deformation can be divided into elastic deformation, plastic deformation and viscous deformation according to the stress–strain behavior.[5]

Elasticity: The ability of an object to deform when resisting an external force and to return to its original size and shape once the force is removed is called elasticity. The resulting deformation is called elastic deformation. Objects with elasticity are called elastic bodies, and they can be divided into two types according to their stress–strain behavior: linear elastic body (ideal elastic body) whose stress–strain behavior is linear and non-linear elastic body whose stress–strain behavior is non-linear.

Plasticity: The ability of an object to deform when resisting an external force and its failure to completely return to its original size and shape after unloading is called plasticity. The deformed portion that cannot return presents plastic deformation or permanent deformation. An object that only undergoes plastic deformation under an external force is called an ideal plastic body. The stress–strain curve of the ideal plastic body is a straight horizontal line. When the stress is lower than the yield limit σ_y, the material is not deformed. After the stress reaches σ_y, the deformation increases, and the internal stress remains unchanged.

Viscosity: The ability of an object to deform under stress and for which the deformation cannot be completed instantly while its strain rate increases with the increase of stress is called viscosity. The stress–strain rate curve of an ideal viscous object (Newtonian fluid) is a straight line passing through the origin point.

In addition to mineral composition and structure, the mechanical properties of rocks are also related to environmental factors, such as stress conditions and temperature. Under normal temperature and pressure, rocks are neither ideal elastic bodies nor simple plastic bodies or viscous bodies. Rocks often present elastic–plastic, plastic–elastic, elastic–viscose–plastic or viscoelastic properties.

2.2.1 Stress–strain relationship

Figure 2.1 depicts the complete stress σ–strain ε curve of a rock specimen under a uniaxial compression load. Full details about the test operations are given in GB/T 50266-2013 and ASTM D7012-14.

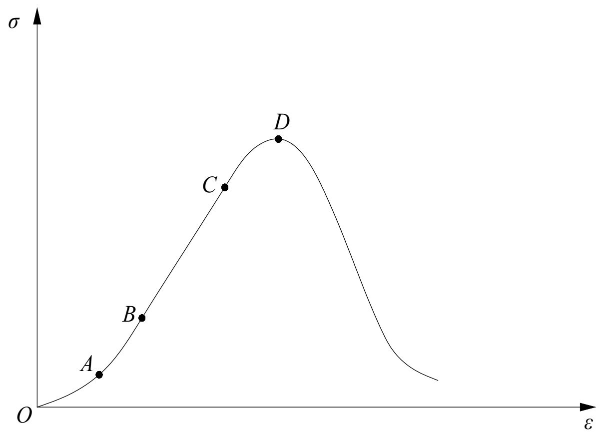


Figure 2.1 Typical curve of stress vs. strain of rock[5]

According to the complete stress–strain curve, the deformation of a rock can be divided into the following four stages:[2,6]

(1) Fissure closing (Section OA): The original open structural surface or micro-cracks in the sample are gradually closed, the rock is compacted, nonlinear deformation occurs, and the stress–strain curve is concave. At this stage, the lateral expansion of the sample is minimal, and its volume decreases as the load increases. The deformation of a fractured rock is more obvious at this stage, whereas the deformation of a hard and less fractured rock is not as obvious.
(2) Elastic compression (Section AC): The stress–strain curve at this stage is at an approximately linear rate. Section AB represents elastic deformation, and BC depicts the stable development of micro-cracks.
(3) Pore structure collapse (Section CD): Point C is the turning point of the rock, or its yield point, from elastic to plastic deformation. The corresponding stress is called yield stress or yield limit. Such stress is two-thirds of the peak strength. After entering this stage, the development of micro-fractures changes qualitatively, and the rupture continues to increase until the sample is completely destroyed. The volume of the sample begins to increase, and the axial strain and volume strain rate increase rapidly. The upper bound stress at this stage is called the peak compressive stress or peak strength of the rock.
(4) Locking (after Point D): The internal structure of the rock block is destroyed, and the sample remains basically intact. At this stage, the cracks develop rapidly, intersect and mutually form a macroscopic fracture surface. Subsequently, the rock block deformation mainly manifests as a block slip along the macro-fracture surface. The bearing capacity of the sample then decreases rapidly with the increase of the deformation, and it generally does not decrease to zero. Thus, the fractured rock block still has a certain bearing capacity.

The stress–strain curves of rocks vary according to their type and nature. Miller divided rocks into six types according to their stress–strain curve characteristics before the peak (Figure 2.2).[6]

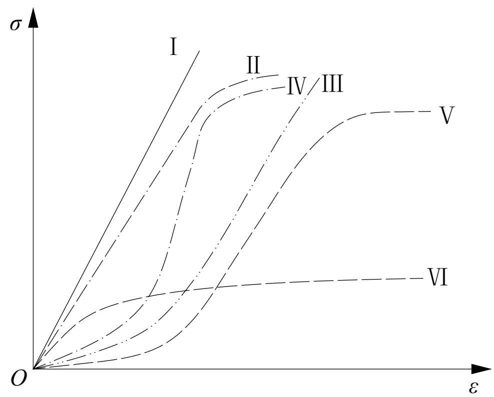


Figure 2.2 Typical rock stress–strain curve before peak strength

Type I: The relationship between pressure and strain is represented as a straight line or until the specimen suddenly breaks. Rocks with such deformation properties (including basalt, quartzite, dolomite and extremely strong limestone) are classified as elastic rocks.

Type II: When the pressure is low, the stress–strain curve is approximately a straight line. When the stress increases to a certain value, the stress–strain curve bends downward, and as the stress increases gradually, the slope of the curve becomes smaller until it is destroyed. Rocks with such deformation properties (including weaker limestone, mudstone and tuff) are called elastic–plastic rocks.

Type III: When the pressure is low, the stress–strain curve bends slightly upward. When the stress increases to a certain value, the stress–strain curve gradually becomes a straight line until damage occurs. Rocks with such deformation properties are called plastic–elastic rocks.

Type IV: When the pressure is low, the stress–strain curve bends upward. When the stress increases to a certain value, the deformation curve becomes a straight line, and the curve bends downward and ultimately shows an S-shaped line. Rocks with such deformation characteristics (including marble and gneiss) are called plastic–elastic–plastic rocks.

Type V: The rock is basically the same as Type IV, and the curve is also an S-shaped line, but the slope of the curve is relatively flat and occurs mostly in rocks with high compressibility, such as schist with stress perpendicular to it.

Type VI: The stress–strain curve begins with a small straight line, follows an inelastic curve portion and continues to creep. Such rocks (including rock salt and soft rock) are called elastic–viscous rocks.

2.2.2 Application of stress–strain curves

In addition to reflecting the constitutive relationships of rocks, the complete stress–strain curve is also applied to the situations listed here:

(1) Prediction of rock burst

As shown in Figure 2.3, the area of the complete stress–strain curve is bounded by the peak strength point C, and the left half of OEC (Area A) represents the strain energy accumulated

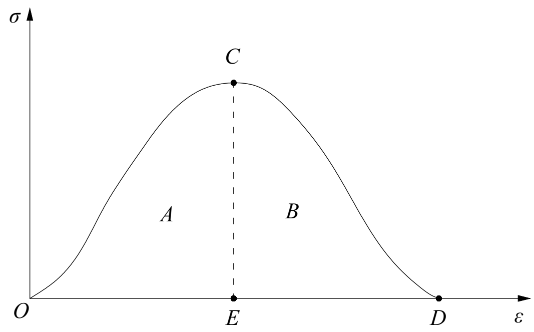


Figure 2.3 Full stress–strain curve used for predicting rock burst[5]

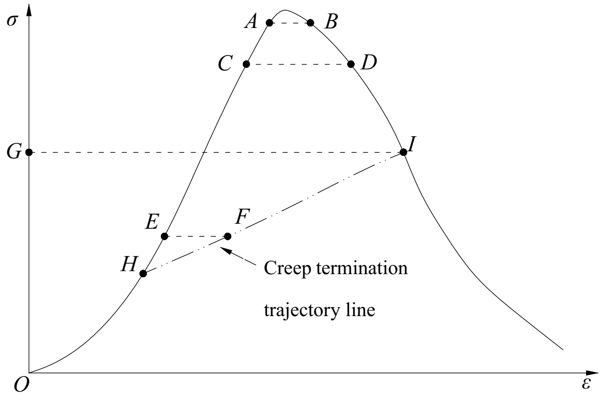


Figure 2.4 Full stress–strain curve for predicting creep damage[2, 6]

in the specimen when the rock reaches peak strength. The right half of DEC (Area B) depicts the energy released by the rock during the destruction process. If B<A, then the rock still has energy remaining after the damage. The rapid release of this part of the energy causes rock burst. If B>A, then the rock has completely released the absorbed strain energy during the failure process, and no rock burst occurs.

(2) Prediction of creep failure

The creep termination trajectory in Figure 2.4 indicates that after the specimen is loaded to a certain stress level, the stress is kept constant, and the specimen creeps. At the appropriate stress level, the creep develops. When the strain reaches a certain value, the creep stops, and the rock specimen reaches a stable state. The creep termination trajectory is the connection of the creep termination point at different stress levels and can be determined by numerous experiments in advance. When the stress level is below point *H*, the stress is kept constant, and the rock specimen does not creep. When the stress level reaches point *E*, the stress is kept constant, and the creep strain develops until point *F* intersects with the creep end trajectory, and the creep stops. Point *G* is the critical point. The stress level remains constant below point *G*, the creep strain develops to intersect the creep termination trajectory, the creep stops, and the rock specimen is not destroyed. If the stress level remains constant at point G, the creep strain develops to the end, and the right half of the stress–strain curve intersects. That is, the broken curve intersects. At this time, the specimen is destroyed. The maximum creep strain value can be obtained. The stress level remains constant above point G and creeps, eventually leading to damage, because at the end, it must intersect the post-destruction segment of the full stress–strain curve. The higher the stress level, the shorter the time from creep to failure, such as the creep from point C to point D and the creep from point A to point B.

(3) Rock damage prediction under a cyclic loading condition

Cyclic loading is often encountered in geotechnical engineering. For example, repeated blasting operations involve applying cyclic loads to surrounding rocks and are dynamic. As most rocks are not linear elastic materials, their loading and unloading paths do not coincide, and each add–unload creates a hysteresis loop, thereby leaving a permanent deformation. Figure 2.5 shows the cyclic loading at high stress levels and the rock breaking shortly. If a cyclic load is applied from point A, then the permanent deformation develops to point B, the peak is intersected with the peak of the full stress–strain curve, and the rock is destroyed. Thus, when the rock engineering itself is in a state of high stress and when the cyclic load occurs again, the rock engineering becomes vulnerable to damage. If subjected to cyclic loading at the stress level at point C, then the loading can be experienced for a relatively long period of time before the rock engineering can be destroyed. Therefore, according to the existing stress level of the rock itself and the magnitude and period of the cyclic load, the complete stress–strain curve is applied to predict the time of rock failure under cyclic loading conditions.

2.2.3 Rock deformability

According to the stress–strain curve of rocks, the deformation parameters such as the deformation modulus and Poisson's ratio of rocks can be determined.

The deformation modulus E is the ratio of the axial compressive stress to the axial strain under uniaxial compression, which is $E=\sigma/\varepsilon$, where σ and ε are the axial stress and strain at any point on the stress–strain curve, respectively.

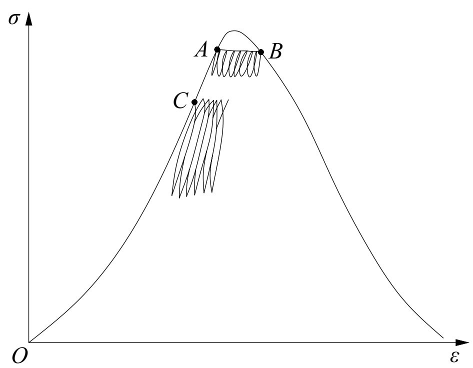


Figure 2.5 Full stress–strain curve predicts failure under cyclic loading conditions[5]

When the stress–strain curve is a straight line, the deformation modulus of the rock is a constant and is numerically equal to the slope of the straight line. As the deformation is mostly elastic deformation, it is also called elastic modulus.

When the stress–strain curve is non-linear, the deformation modulus of the rock is a variable, that is, the modulus on differentiated stress segments. Commonly used examples are the initial modulus E_0, tangent modulus E_t and secant modulus E_s (Figure 2.6).

The initial modulus is the tangent slope at the origin on the curve. The tangent modulus is the slope of the tangent at any point on the curve. The secant modulus refers to the slope of the line connecting the starting point of the curve to another point. The slope of the line connecting the point at $\sigma_c/2$ and the starting point is usually taken.

Poisson's ratio μ is the ratio of the transverse strain ε_x to the longitudinal strain ε_y under uniaxial compression.

$$\mu = \left|\frac{\varepsilon_x}{\varepsilon_y}\right|. \tag{2.22}$$

In practical situations, ε_x and ε_y at $\sigma_c/2$ are commonly used to calculate the Poisson's ratio of rocks.

In addition to the two parameters of elastic modulus and Poisson's ratio, other parameters that reflect the deformation properties of rocks include shear modulus G, Lame constant λ and bulk modulus K_v. These parameters have the following relationship with elastic modulus E and Poisson's ratio μ:

$$G = \frac{E}{2(1+\mu)}, \tag{2.23}$$

$$\lambda = \frac{E\mu}{(1+\mu)(1-2\mu)}, \tag{2.24}$$

$$K_v = \frac{E}{3(1-2\mu)}. \tag{2.25}$$

The elastic parameters described in Equations 2.23, 2.24 and 2.25 are static elastic parameters, that is, the elastic parameters of the material under static load. The elastic

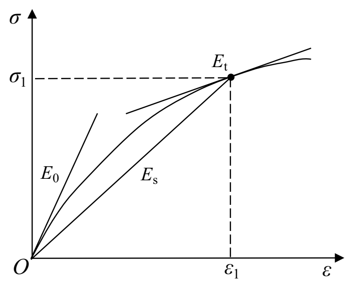


Figure 2.6 Initial, tangent and secant moduli

parameters of the material subjected to dynamic load include rock dynamic elastic modulus E_d, dynamic Poisson's ratio μ_d, dynamic shear modulus G_d, dynamic pull constant λ_d and dynamic bulk modulus K_d. These parameters can be calculated from the longitudinal and transverse wave velocities of rocks. Full details about the rock sound wave velocity test are given in GB/T 50266-2013.

2.3 Rock strengths

Rock strengths commonly adopted in engineering practice include Uniaxial Compressive Strength, point load strength, tensile strength and triaxial compressive strength.

2.3.1 Uniaxial compressive strength

The uniaxial compressive strength (UCS) of rocks σ_c is the maximum compressive stress that rocks can withstand under uniaxial (unconfined) compression conditions. During the test, the specimen is subjected only to a vertical pressure; the lateral pressure is unrestrained, and the deformation of the specimen is not limited. The uniaxial compressive strength of rock is a basic index for determining the mechanical properties of rock mass. It is also an important indicator for the engineering classification of rock mass and for establishing the criteria for rock failure. Such strength can also be adopted to estimate the other strength parameters of rocks.

Full details about the test are given in GB/T 50266-2013 and ASTM D7012-14.

Under the condition of uniaxial compression, the test piece mainly produces the following three types of damage:

(1) Shear failure of X-shaped conjugate slope, $\beta \approx \dfrac{\pi}{4} + \dfrac{\phi}{2}$; (Figure 2.7a);

(2) Single level shear failure, $\beta \approx \dfrac{\pi}{4} + \dfrac{\phi}{2}$; (Figure 2.7b);

(3) Lateral tensile failure. Under axial compressive stress, tensile stress is generated in the transverse direction because of the Poisson effect. Such damage occurs when the transverse tensile stress exceeds the tensile strength of rocks (Figure 2.7c).

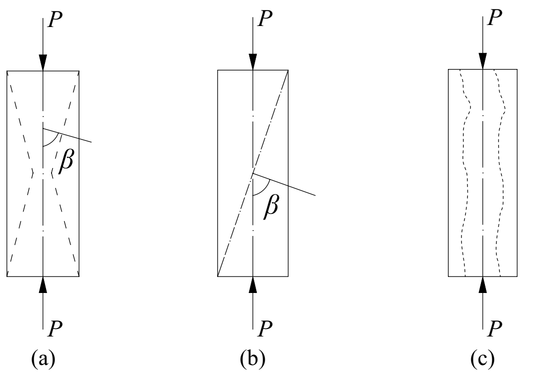


Figure 2.7 Failure states in uniaxial compression test[5]

The first two types of damage are caused by shear stress on the failure surface that exceeds the shear strength, a phenomenon which is generally called compression shear failure. Accordingly, the third type of damage is sometimes referred to as compression failure.

2.3.2 Point load strength

The test for determining the point load strength of rocks is a simple rock strength test proposed by E. Broch and J. A Franklin in 1972. This index can be referred to in particle distribution and engineering design. Full details about the point load test are given in GB/T 50266-2013 and ASTM D5731-16.

The point load strength index is usually presented as I_s and is expressed as

$$I_s = \frac{P}{D_e^2}, \tag{2.26}$$

where P is the load at which the specimen is broken (N), D_e is the diameter of the equivalent rock core (mm), $D_e = \sqrt{\frac{4A}{\pi}}$ and A is the minimum cross-sectional area (mm^2) between the points of loading.

In practice, the core diameter exerts a major impact on the value of the point load strength. The core diameter is changed from 10 mm to 70 mm, and the point load strength can be reduced by 2 to 3 times. Accordingly, the International Society of Rock Mechanics determines the strength index value $I_s(50)$ of the radial load point load test of a 50 mm cylindrical specimen as a standard test value. The point load strength index can also be converted with the Uniaxial Compressive Strength. The conversion equation is

$$\sigma_c = 24 I_s(50). \tag{2.27}$$

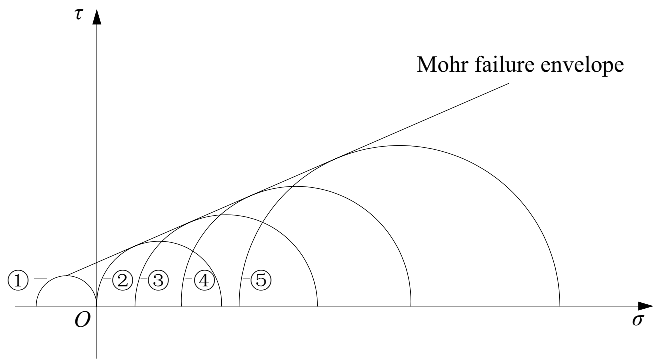
2.3.3 Tensile strength

The tensile strength of rocks is the maximum tensile stress that rocks can withstand when they are damaged by uniaxial tensile load. It is expressed by σ_t and can be adopted to determine the strength envelope and establish a rock strength criterion. Tensile strength can be identified directly by stretching or indirectly by the Brazilian test. Because of the swiftness and convenience of the Brazilian test, it has become a common method for determining the tensile strength of rocks. Full details about the test are given in GB/T 50266-2013 and ASTM D3967-16. Such tensile strength can be expressed as

$$\sigma_t = \frac{2P}{\pi d t}, \tag{2.28}$$

Where σ_t is the tensile strength (Mpa), P is the compressive load (MPa), d is the diameter of the cylinder (mm) and t is the height of the cylinder (mm).

In general, the short cracks present in rocks considerably influence the direct drawing method rather than the Brazilian test method. Thus, the strength value given by the Brazilian



① Mohr circle for specimen failure under uniaxial tensile test
② Mohr circle for specimen failure under uniaxial compression test
③④⑤ Mohr circle for specimen failure during three axial compression test

Figure 2.8 Strength envelope

test is usually higher than that of the direct tensile test, and the difference can sometimes be several times greater.

2.3.4 Shear strength

When the confining pressure is zero or low, a rock usually breaks along a set of inclined fissures. Under high confining pressure, the ductile deformation and strength of the rock increase, and full ductility or plastic flow deformation occurs, along with hardening. The test specimen also exhibits a thick waist barrel-like shape. Full details of the triaxial tests are given in GB/T 50266-2013 and ASTM D7012-14.

The triaxial compression test is mainly used to determine the Mohr strength of rocks. The Mohr strength envelope (Figure 2.8) can be identified through triaxial compression on several specimens in the same coordinate system, drawing the stress state and Mohr circle at the time of the failure of each specimen and supplementing the Mohr circle at the time of the uniaxial compression test and tensile failure test.

2.4 Theory of rock strength

When a rock is destroyed, its stress and strain present a certain relation. The functional relation characterising the stress and strain of rock failure is called the failure criterion or strength criterion. The failure modes of rocks are mainly divided into brittle fracture and ductile fracture. Hard rocks are chiefly characterised by brittle failure in low confining pressure and low temperature environment. Note that hard rocks tend to exhibit ductile failure under high confining pressure and high temperature environment (Table 2.2).

The strength theory of rock mechanics includes maximum positive strain theory, Mohr strength theory, Griffith strength theory and strain energy theory (Table 2.3). Given their wide applicability, Mohr's strength theory and Griffith strength theory are mainly discussed in this work.

Table 2.2 Failure modes of rock under triaxial compression[6]

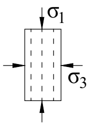
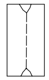
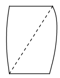
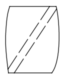
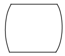
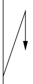
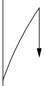
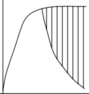
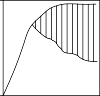
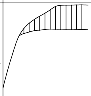
Strain at failure / %	<1	1~5	2~8	5~10	>10
Destructive form	Brittle failure	Brittle failure	Transition failure	Ductile failure	Ductile failure
Failure of specimens					
Basic type of stress-strain curve					
Failure mechanism	Tension rupture	Rupture with tension	Shear rupture	Shear flow rupture	Plastic flow

Table 2.3 Comparison of the theories of rock strength

Strength theory	*Theoretical expression*	*Failure condition*	*Scope of application*
Maximum normal strain theory	Rupture of the object occurs because the extension strain ε reaches the ultimate strain ε_0.	$\lvert\sigma_3-\mu(\sigma_1+\sigma_2)\rvert\geq\sigma_t$ $\sigma_1\geq\frac{1-\mu}{\mu}\sigma_3+\sigma_c$ σ_c is the rock's uniaxial tensile strength, σ_t is the rock's tensile strength, and μ is the Poisson's ratio.	Only for brittle rocks
Mohr Coulomb strength theory	When a certain functional relationship between shear stress and normal stress on one surface is satisfied, the surface breaks.	$\tau\geq\sigma\tan\varphi+c$ $\sin\varphi\leq\frac{\sigma_1-\sigma_3}{\sigma_1+\sigma_3+2c\cdot\cot\varphi}$ φ is the internal friction angle, and c is the cohesion.	Suitable for rock damage under low confining pressure conditions.
Griffith's strength theory	When the maximum stress on the cracked side wall of the rock is greater than the tensile strength of the rock, the crack expands.	(1) When $\sigma_1+3\sigma_3\geq0,$ $\frac{(\sigma_1-\sigma_3)^2}{\sigma_1+\sigma_3}\geq-8\sigma_t.$ (2) When $\sigma_1+3\sigma_3<0,$ $\lvert\sigma_3\rvert\geq\lvert\sigma_t\rvert.$ σ_t is the rock's tensile strength.	Suitable for the tensile failure of brittle rocks
Shear strain energy theory	Rock failure is due to the plastic flow caused by the shear strain in the unit volume reaching the limit value.	$\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]\geq\sigma_y{}^2.$ σ_y is yield strength of the material.	Suitable for rocks with ductile failure

2.4.1 Mohr criterion

In 1900, Mohr proposed that the destruction of a material mainly involves shear failure and that the shear stress on the failure surface is a function of the normal stress on that surface. Mohr also made some assumptions about the characteristics of rock failure. He contended that rock strength is independent from the magnitude of the intermediate principal stress σ_2. In addition, the macroscopic fracture surface of a rock is basically parallel to the direction of the intermediate principal stress. Mohr circle can be drawn in a rectangular coordinate system in which the shear stress τ is the vertical axis and normal stress σ is the horizontal axis. In this coordinate axis and with an infinite number of ultimate stress circles, the trajectory line of the breaking stress point is called the Mohr strength line or the Mohr envelope.

As a result of the obvious heterogeneity of rocks, the calculation formula of the Mohr envelope can only be represented with a universal function

$$\tau = f(\sigma). \tag{2.29}$$

The curve of rock strength can be determined by the triaxial compression test. Full details about the test are given in GB/T 50266-2013. If the stress state at any point in the rock falls below the Mohr strength envelope, then the rock will not be destroyed (Case 1, Figure 2.9). If the stress state is tangent to the Mohr strength envelope, then the rock will undergo shear damage (Case 2 in Figure 2.9). If the stress state intersects the Mohr strength envelope, then the rock will be destroyed (Case 3, Figure 2.9).

The Mohr strength envelope is usually presented in three forms: linear envelope, hyperbolic criterion and parabolic criterion.

(1) Linear envelope

The linear strength envelope is also known as the Mohr-Coulomb criterion (Figure 2.10). The Mohr-Coulomb criterion characterises the shear failure of rocks and is suitable for low confining pressures. The angle φ between the line and the σ axis is called the angle of internal friction, and the intercept on the τ axis is the adhesion force c and is expressed as

$$\tau = \sigma \tan \varphi + c. \tag{2.30}$$

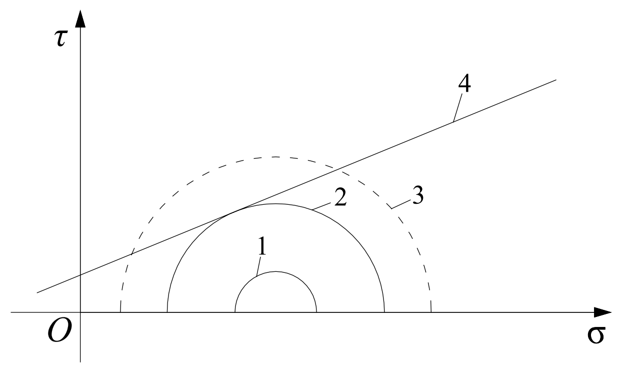


Figure 2.9 Determination of material failure with Mohr envelope 1 – unbroken; 2 – critical; 3 – failed; and 4 – strength envelope.

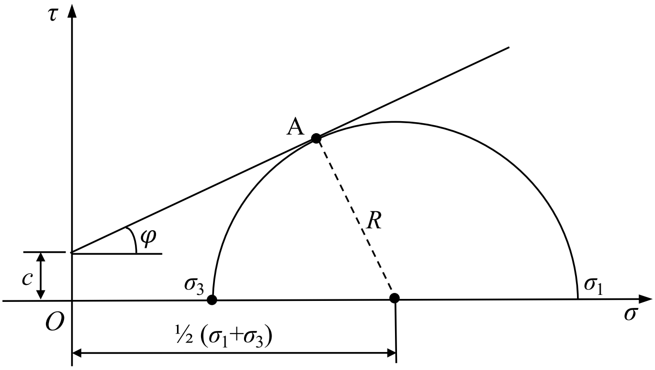


Figure 2.10 Mohr-Coulomb strength

If we assume that the maximum principal stress circle of a micro-body in a rock sample is tangent to the Mohr-Coulomb strength envelope, then such envelope can be ascertained from the geometric relationship shown in Figure 2.10.

$$\sin\varphi = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3 + 2c \cdot cot\varphi}. \tag{2.31}$$

Thus, the conditions for determining the failure of rocks using the Mohr-Coulomb strength criterion are

$$\sin\varphi \le \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3 + 2c \cdot cot\varphi} \tag{2.32}$$

or

$$\tau \ge \sigma \tan\varphi + c. \tag{2.33}$$

From Formula (2.31), we can obtain

$$\sigma_1 = \sigma_3 \frac{1 + sin\varphi}{1 - cos\varphi} + \frac{2c \cdot cos\varphi}{1 - sin\varphi} \tag{2.34}$$

Under axial compression, σ_3=0, and from Formula (2.31),

$$\sigma_1 = \frac{2c \cdot cos\varphi}{1 - sin\varphi}. \tag{2.35}$$

Thus, the uniaxial compressive strength $\sigma_c = \frac{2c \cdot cos\varphi}{1 - sin\varphi}$, setting $\frac{1 + sin\varphi}{1 - cos\varphi} = \xi$, leads Formula (2.31) to be expressed with principle stress as

$$\sigma_1 = \sigma_3 \xi + \sigma_c. \tag{2.36}$$

(2) Hyperbolic criterion

The hyperbolic strength criterion curve is applied to hard rocks such as sandstone, limestone and granite. It can be expressed as

$$\tau^2 = (\sigma + \sigma_t)^2 \tan^2\eta + (\sigma + \sigma_t)\sigma_t, \tag{2.37}$$

where η is the inclination of the asymptote of the envelope (Figure 2.11) and $\tan \eta = \frac{1}{2}\left(\frac{\sigma_c}{\sigma_t} - 3\right)^{1/2}$.

The criterion for rock damage is

$$\tau^2 \geq (\sigma + \sigma_t)^2 \tan^2\eta + (\sigma + \sigma_t)\sigma_t. \tag{2.38}$$

The calculation formula of tan η indicates that when $\sigma_c/\sigma_t < 3$, tan η will have a void value. Thus, this model does not apply to rocks with $\sigma_c/\sigma_t < 3$.

(3) Parabolic criterion

The quadratic parabolic strength criterion curve is applicable to rocks with weak lithology, such as mudstone and shale. It is expressed as

$$\tau^2 = n(\sigma + \sigma_t). \tag{2.39}$$

where σ_t is the uniaxial compressive strength of the rock and n is the undetermined coefficient.

According to the geometric relationship shown in Figure 2.12, Formula (2.39) can be converted into

$$(\sigma_1 - \sigma_3)^2 = 2n(\sigma_1 + \sigma_3) + 4n\sigma_t - \mathrm{n}^2. \tag{2.40}$$

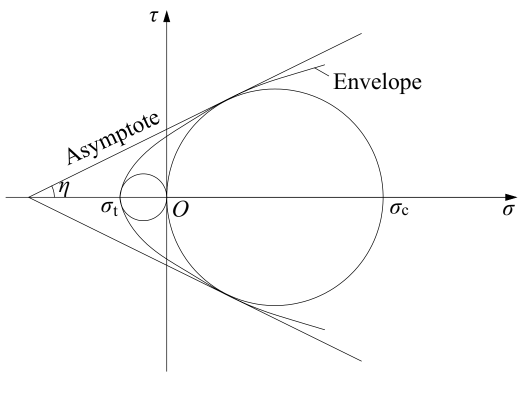


Figure 2.11 Hyperbolic strength[6]

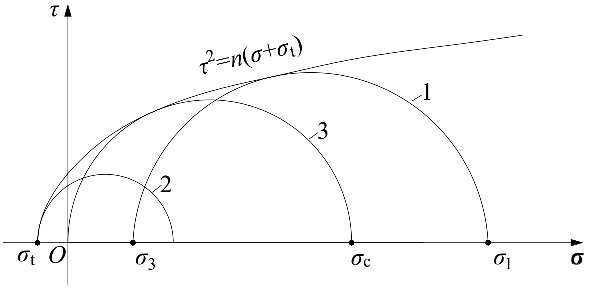


Figure 2.12 Quadratic parabolic strength

1 – Any triaxial compression stress circle

2 – Tensile stress circle

3 – Uniaxial compressive stress circle

Under uniaxial compression, $\sigma_3 = 0$, $\sigma_1 = \sigma_c$, and Formula (2.40),

$$n = \sigma_c + 2\sigma_t \pm 2\sqrt{\sigma_c\left(\sigma_c + \sigma_t\right)} \tag{2.41}$$

By substituting Formula (2.41) into Formula (2.39), the criterion for rock damage is expressed as

$$\tau^2 \geq \left(\sigma_c + 2\sigma_t \pm 2\sqrt{\sigma_c\left(\sigma_c + \sigma_t\right)}\right)\cdot\left(\sigma + \sigma_t\right) \tag{2.42}$$

2.4.2 Griffith theory

Rocks are not an ideal continuous medium, and they contain numerous micro-fissures. Griffith believed that when a brittle material is stressed, the end of the internal micro-fissures generates concentrated stress. When the concentrated stress exceeds the tensile strength of the material, the micro-fissures begins to expand, combine and accommodate. Finally, a macroscopic rupture appears along one or several planes or on the curved surfaces of the rock.

Griffith's theory assumes that the fissures in a rock are flat and elliptical and that they do not affect one another (Figure 2.13).

From Figure 2.13, the tensile stress generated near the internal fissure tip is

$$\sigma_b m = \sigma_y \pm \left(\sigma_y^2 + \tau_{xy}^2\right)^{1/2}, \tag{2.43}$$

where σ_b is the maximum tangential stress around the ellipse, coefficient m is calculated as $m = b/a$ (a is the major semi-axis of the ellipse, and b is the minor semi-axis), σ_y is the normal stress in the direction of vertical fissures and τ_{xy} is the shear stress in the direction parallel to the fissures.

When the tangential stress σ_b around the fissures reaches or exceeds the tensile strength σ_t of the rock, the fissures begin to expand. The σ_t and m of the sidewall of the fissures are difficult to determine. Accordingly, the simplest case can be assumed, that is, the long axis

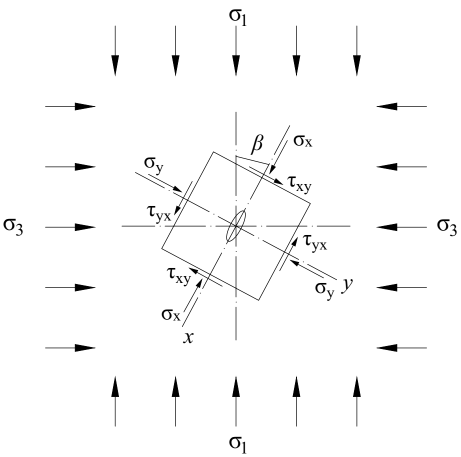


Figure 2.13 Stresses on the periphery of a unit with a crack inclined at an angle β to the maximum principal stress[3]

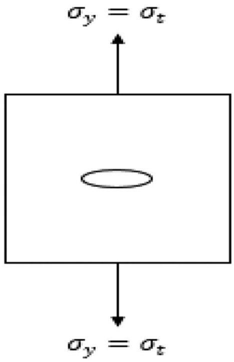


Figure 2.14 Fissure propagation during uniaxial tension

of the fissures is perpendicular to the uniaxial tensile stress to determine the critical value in the tangential stress (Figure 2.14).

(1) The strength criterion can be expressed with σ_y and τ_{xy}:

When the rock is uniaxially stretched to damage, $\tau_{xy} = 0$, $\sigma_y = \sigma_t$, from Formula (2.43). Then,

$$\sigma_b m = \sigma_t + \sigma_t = 2\sigma_t. \tag{2.44}$$

Thus, the strength criterion can be expressed as

$$2\sigma_t \le \sigma_y \pm \left(\sigma_y^2 + \tau_{xy}^2\right)^{1/2}. \tag{2.45}$$

This formula is the Griffith strength criterion, that is, the relation between the normal stress σ_y and the shear stress τ_{xy} when the fissure starts to expand.

(2) The strength criterion can be expressed with σ_1 and σ_3:
σ_y and τ_{xy} are replaced with σ_1 and σ_3, respectively. Then,

1) When $\sigma_1 + 3\sigma_3 \geq 0$, crack failure angle β is

$$\cos 2\beta = \frac{\sigma_1 - \sigma_3}{2(\sigma_1 + \sigma_3)}. \tag{2.46}$$

Then, the Griffith strength criterion can be expressed as

$$\frac{(\sigma_1 - \sigma_3)^2}{\sigma_1 + \sigma_3} \geq -8\sigma_t. \tag{2.47}$$

2) When $\sigma_1 + 3\sigma_3 < 0$, crack failure angle β is

$$\beta = 0.$$

Then, the Griffith strength criterion can be expressed as

$$|\sigma_3| \geq |\sigma_t|. \tag{2.48}$$

According to this criterion, the following formula can be obtained under the uniaxial stress σ_3=0 from (2.47):

$$\sigma_c = -8\sigma_t. \tag{2.49}$$

The Griffith strength criterion is proposed for brittle materials such as glass and steel, and it is therefore only suitable for studying the destruction of brittle rocks. For general rock materials, the Mohr-Coulomb strength criterion is much more applicable than the Griffith counterpart.

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