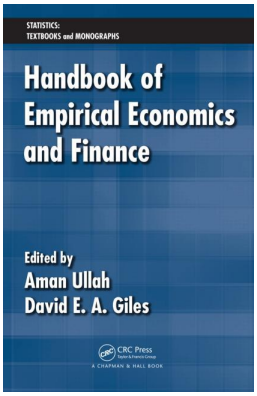

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Structural Macroeconometric Modeling in a Policy Environment

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9

Structural Macroeconometric Modeling in a Policy Environment

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CONTENTS

9.1 Introduction

Since the basic ideas of structural macroeconometric modeling were laid out by the Cowles Commission, there has been substantial effort invested in turning their vision into a practical and relevant tool. Research and development has proceeded across a broad front but basically can be characterized as responses to four issues.

1. The design of models to be used in a policy environment.
2. Estimation of the parameters in these models.
3. Match of these models to the data, i.e., how to evaluate their ability to adequately represent the outcomes from an actual economy.
4. Prediction and policy analysis with the models.

Econometric texts and articles typically deal with the last three topics while the first tends to be neglected. Consequently this chapter will focus on how the design of models in policy use has evolved over the past 60 years. In concentrating on the models that have been adopted in institutions concerned with policy making, we have not dealt with models either developed in the private sector or by individuals, e.g., the model set out initially by Fair (1974) which has gone through several generations of change. Moreover, although our primary focus is on model design, it is impossible to ignore questions of estimation and data matching, as often these are driven by the design of the models, so that we will need to spend some time on the second and third of the issues.

Model design has evolved in a number of ways. At a primal level it is due to the fact that the academic *miniature* model upon which they are based, and which aims to capture the essential forces at work in the economy, has changed over time. We can distinguish five of these miniature models:

1. Ramsey model – Ramsey (1928)
2. IS-LM, Aggregate Demand-Supply (AD-AS) models – Hicks (1937).
3. Solow-Swan Model – Solow (1956), Swan (1956)
4. Stochastic Ramsey Model (Real Business Cycle Model/Dynamic Stochastic General Equilibrium -DSGE- models) – King Plosser, and Rebelo (1988)
5. New Keynesian model – Clarida, Gali, and Gertler (1999)

Essentially, these models were meant to provide a high-level interpretation of macroeconomic outcomes. Mostly they were too simple for detailed policy work and so needed to be *adapted* for use. Although providing some broad intellectual foundations they need to be augmented for practical application. The adaptions have led to four generations of models distinguished later which loosely relate to the miniature models given above.

Coexisting with these *interpretative models* have been *summative* models that aim to fit a given set of data very closely and which employ various statistical approaches to do this, e.g., vector autoregressions (VARs). Mostly these models are used for forecasting. Sometimes the summative and interpretative models have been identical, but increasingly there has been a divorce between them, resulting in a multiplicity of models in any policy institution today. To some extent this reflects developments in computer hardware and software since the cost of maintaining a variety of models has shrunk quite dramatically in the past few decades. The greater range of models also means that how we are to judge or evaluate a given model will differ depending upon what it seeks to achieve. Consequently, this often accounts for why

proponents of a particular representative of each of the classes are reluctant to evaluate their models with criteria that might be appropriate for another of the classes.

The four generations of models we will distinguish in the succeeding sections are often represented as being vastly different. Sometimes the differences that are stressed are superficial, reflecting characteristics such as size and underlying motivation. It would be unfortunate if this attitude prevailed as it obscures the fact that each generation has drawn features from previous generations as well as adding new ones. Evolution rather than revolution is a better description of the process describing the move from one generation to another. To see this it will help to structure the discussion according to how each generation has dealt with five fundamental questions:

1. How should the dynamics evident in the macroeconomy be incorporated into models? Specifically, are these to be external (imposed) or internal (model consistent)?
2. How does one incorporate expectations and what horizon do they refer to?
3. Do stocks and flows need to be integrated? If so, is this best done by having an equilibrium viewpoint in which all economic variables gravitate to a steady-state point or growth path?
4. Are we to use theoretical ideas in a loose or tight way?
5. How are nominal rather than real quantities to be determined?

The sections that follow outline the essential characteristics of each of the four generations of models distinguished in this chapter by focusing on the questions just raised. This enables one to see more clearly what is common and what is different between them.

9.2 First Generation (1G) Models

These are the models of the 1950s and 1960s. If one had to associate a single name with them it would be Klein. If one had to associate a single institution it would be the University of Pennsylvania. A very large number of modelers in many countries went to the latter and were supervised by the former.

The miniature model that underlies representatives of this generation was effectively that associated with the IS/LM framework. Accordingly, the modeling perspective was largely about the determination of demand. Adaption of the miniature model to policy use involved disaggregation of the components of the national income identity. Such a disaggregation inevitably led to these models becoming large.

Dynamics in the models were of two types. One alternative was to allow for a dynamic relation between y_t and x_t by making y_t a function of $\{x_{t-j}\}_{j=0}^{p}$. If p was large, as might be the case for the effect of output (x_t) upon investment (y_t), then some restrictions were imposed upon the shape of the

lagged effects of a change in x_t upon y_t. A popular version of this was termed "Almon lags"– Almon (1965). But mostly dynamics were imposed using a different strategy — that associated with the partial adjustment model (PAM). With real variables in logs (some nominal variables such as interest rates, however, were left in levels form) this had the structure[1]

$$\Delta z_t = \gamma(z_t^* - z_{t-1}), \tag{9.1}$$

where z_t^* was some target for z_t which was made observable by relating it to a function of x_t. The specification of the function linking z_t^* and x_t was generally loosely derived from theoretical ideas. As an example, targeted consumption c_t^* was related to income (y_t) and other things expected to influence consumption, such as interest rates (r_t). Thus,

$$c_t^* = a y_t + b r_t. \tag{9.2}$$

In these models there was often an awareness of the importance of expectations in macroeconomics, reflecting their long history in macroeconomic discussion. To model these expectations, one assumed they could be measured as a combination of the past history of a small set of variables (generally) present in the model, with the weights attached to those variables being estimated directly using the observations on the variables expectations were being formed about.

Because the supply side in these models was mostly ignored, there was not a great deal of attention paid to stocks and flows. Wallis (1995), in an excellent review of these and the 2G models discussed later, notes that there was an implicit assumption underlying them that variables evolved deterministically over longer periods of time, although there was not any discussion about whether such paths were consistent and their relative magnitudes did not seem to play a major role in model construction and design.

To build a link between the real and nominal sides of the economy modelers generally viewed prices as a mark up over (mostly) wages, and the markup was often influenced by business conditions. A dynamic account of wages was provided by the Phillips curve. Later versions just assumed that the Phillips curve applied to inflation itself and so had the form

$$\pi_t = \alpha_1 \pi_{t-1} + \delta u_t + \varepsilon_t, \tag{9.3}$$

where π_t was price inflation and u_t was the unemployment rate. There was a lot of debate about whether there was a trade-off between inflation and unemployment, i.e., was $\delta \neq 0, \alpha_1 < 1$? Sometimes one saw this relation augmented as

$$\pi_t = \alpha_1 \pi_{t-1} + \delta(u_t - \overline{u}) + \gamma_2(p_{t-1} - ulc_{t-1}) + \varepsilon_t, \tag{9.4}$$

[1] In many of the early models variables were expressed in terms of their levels and it was only later that log quantities were used more extensively.

where p_t was the log of the price level and ulc_t was unit labor cost. Without some modification like this there was no guarantee that the level of prices and wages would remain related.

Estimation of these models was mostly done with single equation methods and so evaluation largely involved applying a range of specification tests to the individual equations. These equations could be represented as

$$y_t = \phi_1 y_{t-1} + \phi_2 z_t + \phi_3 z_{t-1} + \varepsilon_t, \tag{9.5}$$

where z_t might be endogenous variables and ε_t was an "error term." Tests therefore considered the residuals $\hat{\varepsilon}_t$ as a way of gaining information about specification problems with this equation. Although useful, this evaluation process did not tell one much about the fit of the complete model, which was a key item of interest if the model is to be used for forecasting. For that it needs to be recognized that z_t is not given but also needs to be solved for. System and single equation performance might therefore be very different.

Once a complete system was found one could find a numerical value for what one would expect z_t to be from the model (given some exogenous variables) either analytically or by simulation methods (when the system was nonlinear). The software developed to do so was an important innovation of this generation of models. Chris Higgins, one of Klein's students, and later Secretary of the Australian Treasury, felt that any assurance on system performance required that modelers should "simulate early and simulate often." For that, computer power and good software were needed. It was also clear that, in multistep forecasts, you had to allow for the fact that both y_{t-1} and z_{t-1} needed to be generated by the model. Hence dynamic simulation methods arose, although it is unclear if these provided any useful extra information about model specification over that available from the static simulations, since the residuals from dynamic simulations are just transformations of the $\hat{\varepsilon}_t$.[2] Perhaps the major information gained from a dynamic simulation of the effects following from a change in an exogenous variable was what happened as the policy horizon grew. If the change was transitory, i.e., lasted for only a single period, then one would expect the effects to die out. In contrast, if it was permanent, one would expect stabilization of the system at some new level. It was easy to check that this held if one only has a single equation, e.g., in the PAM scheme $0 < \gamma < 1$ was needed. Thus each of the individual equations could be checked for stability. But this did not guarantee system stability because, *inter alia*, z_t might depend upon y_{t-1}, thereby making the stability condition much more complex. An advantage of a dynamic simulation was that it could provide the requisite information regarding the presence or absence of stability relatively cheaply and easily.

[2] Wallis (1995) has a good discussion of these issues.

9.3 Second Generation (2G) Models

These began to emerge in the early 1970s and stayed around for 10–20 years. Partly stimulated by inflation, and partly by the oil price shocks of the early 1970s, the miniature model that became their centerpiece was the AD/AS model — which recognized the need for a supply side in the model. When adapted for use this involved introducing a production function to place a constraint on aggregate supply, particularly over longer horizons. A leading light in the development of these models for policy use was John Helliwell with his RDX2 model of the Canadian economy (Helliwell et al. 1971), but others emerged such as the Fed-MIT-Penn (FMP) model (Brayton and Mauskopf 1985) which was also called MPS, see Gramlich (2004).

These models retained much of the structure of the previous generation in that demand was captured by disaggregated equations stemming from the national income identity. Now these were supplemented with equations which introduced much stronger supply side features. There was also some movement toward deriving the relationships as the consequence of optimization problems solved by agents — in particular the consumption decision and the choice of factors of production were often described in this way. Thus for consumption an intertemporal dimension was introduced through the use of life-cycle ideas. These implied that consumption depended on financial wealth (y_t) and current labor income (w_t), i.e., $c_t^* = a w_t + b y_t$. Dynamics were again introduced through a distributed lag on the static relationships determining the desired levels z_t^*. The advance on previous work was the use of an error correction mechanism (ECM),

$$\Delta z_t = \delta \Delta z_t^* + \alpha(z_{t-1} - z_{t-1}^*). \tag{9.6}$$

As Wallis (1995) observes the ECM originated in Phillips' control work of the 1950s and was applied by Sargan (1964) when modeling inflation, but its widespread use began with Davidson et al. (1978).

Now, with the introduction of a production function, and a household's decisions coming loosely from a life cycle perspective, the presence of household wealth and the capital stock meant that there were dynamics present in the model which stemmed from depreciation and savings. Consequently, dynamic stability of the complete system became a pressing issue. Gramlich (1974) comments on his work with the MPS model that "... the aspect of the model that still recalls frustration was that whenever we ran dynamic full-model simulations, the simulations would blow up."Once again one needed to keep an eye on system performance when modifying the individual equations. It might be a necessary condition that the individual equations of the system were satisfactory, but it was not a sufficient one.

Like the previous generation of models there was considerable diversity within this class and it grew larger over time. Often this diversity was the result of a slow absorption into practical models of new features that were becoming important in academic research. For example, since many of these

models had an array of financial assets — certainly a long and a short rate– rational (or model consistent) expectations were increasingly introduced into the financial markets represented in them. By the end of the era of 2G models, this development was widely accepted. But, when determining real quantities, expectations were still mainly formulated in an ad hoc way. One reason for this was the size of the models. The UK models were almost certainly the most advanced in making expectations model-consistent. By 1985 this work had produced a number of models, such as the London Business School and National Institute models, which had implemented solutions; see the review in Wallis and Whitley (1991). A significant factor in this movement was the influence of the Macro-Economic Modelling Bureau at the University of Warwick (see Wallis 1995).

Dynamics in prices were again operationalized through the Phillips curve, but with some modifications. Now either a wage or price Phillips curve had the form

$$\pi_t = \alpha_1 \pi_{t-1} + \delta(u_t - \overline{u}) + \varepsilon_t, \tag{9.7}$$

where $\overline{u}$ was the nonaccelerating inflation rate of unemployment (NAIRU), and, often, $\alpha_1 = 1$. The NAIRU was a prescribed value and it became the object of attention. Naturally questions arose of whether one could get convergence back to it once a policy changed. In models with rational expectations dynamic stability questions such as these assume great importance. If expectations are to be model consistent, then one needed the model to converge to some quantity. Of course one might circumvent this process by simply making the model converge to some prespecified terminal conditions, but that did not seem entirely satisfactory. By the mid 1980s, however, it appeared that many of the models had been designed (at least in the UK) to exhibit dynamic stability, and would converge to a steady state (or an equilibrium deterministic path).

9.4 Third Generation (3G) Models

9.4.1 Structure and Features

Third generation (3G) models reversed what had been the common approach to model design by first constructing a steady-state model (more often a steady-state deterministic growth path, or balanced growth path) and then later asking if extra dynamics needed to be grafted on to it in order to broadly represent the data. Since one of the problems with 2G models was getting stocks to change in such a way as to eventually exhibit constant ratios to flows, it was much more likely that there would be stock-flow consistency if decisions about expenditure items came from well-defined optimization choices for households and firms, and if rules were implemented to describe the policy decisions of monetary and fiscal authorities. In relation to the latter external debt was taken to be a fixed proportion of GDP and fiscal policy

was varied to attain this. Monetary authorities needed to respond vigorously enough to expected inflation — ultimately more than one-to-one to movements in inflation.

There are many versions of 3G models, with an early one being an Australian model by Murphy (1988) and a multi-country model (MSG) by McKibbin and Sachs (McKibbin 1988; McKibbin and Sachs 1989). Murphy's model was more fully described in Powell and Murphy (1995). 3G models became dominant in the 1990s, being used at the Reserve Bank of New Zealand (FPS, Black et al. 1997), the Federal Reserve (FRB-US, Brayton and Tinsley 1996) and, more recently, the Bank of Japan Model (JEM, Fujiwara et al. 2004). Probably the most influential of these was QPM (quarterly projection model) built at the Bank of Canada in the early to mid-1990s, and described in a series of papers (e.g., Black et al., 1994; Coletti et al., 1996). Its steady-state model (QPS) was basically an adaption of the Ramsey model for policy use. To this point in time the latter miniature model had played a major role in theoretical economics but a rather more limited one in applied macroeconomics. An important variation on Ramsey was the use of an overlapping generations perspective that modified the discount rate by the probability of dying, as advocated in Blanchard (1985) and Yaari (1965).

As a simple example of the change in emphasis between 2G and 3G models, take the determination of equilibrium consumption. It was still the case that consumption ultimately depends on financial wealth and labor income, but now the coefficients attached to these were explicitly recognized to be functions of a deeper set of parameters — the steady-state real rate of return, utility function parameters and the discount factor. Because these parameters also affect other decisions made by agents, one cannot easily vary any given relationship, such as between consumption and wealth, without being forced to account for the impact on other variables of such a decision.

Thus a steady-state model was at the core of 3G models. How was it to be used? In a strict steady-state (SSS) dynamics have ceased and values of the variables consistent with these equations will be constant (more generally one could allow for a constant steady-state growth path, but we will leave this qualification for later sections). But the model generating the steady state has embedded in it *intrinsic dynamics* that describe the transition from one steady-state position to another. These dynamics come from the fact that the capital stock depreciates and assets accumulate. Consequently, solving the model produces a *transitional* steady-state solution for the model variables, i.e., these variables will vary over time due to the fact that movements from one point to another are not instantaneous. In addition to this feature, in 3G models some variables were taken to be exogenous, i.e., treated as determined outside the model economy. Since it is unlikely that these will be at their steady-state values over any period of time, the endogenous variable solutions using either the pure or transitional steady-state model will need to reflect the time variation of those exogenous variables. One might refer to the latter values as the *short-run steady-state (SRSS) solutions*.

In adapting the steady-state model for use, it was necessary to recognize that the intrinsic dynamics were rarely sufficient to track the movements of variables in actual economies. Thus it became necessary to augment the intrinsic dynamics. Generally this involved a second stage optimization. The model with the augmented dynamics constituted QPM. The intrinsic dynamics in QPS might therefore be called the *first-stage dynamics*, while the extra dynamics introduced into QPM could be labeled the *second-stage dynamics*. To implement this second stage, one might have simply specified an ECM relating z_t to the SRSS values z_t^* and, in some 3G models, this was how it was done, e.g., Murphy (1988). But in QPM the extra dynamics were introduced in a quasi-theoretical way by choosing z_t to minimize the objective function

$$\frac{1}{2}\sum_{j=0}^{\infty}\beta^j E_t\{(z_{t+j}-z_{t+j}^*)^2+\phi(\Delta z_{t+j}-E(\Delta z_{t+j}))^2\}, \tag{9.8}$$

where $E_t(.)$ is the expected value conditional upon the information available at t. Setting $E(\Delta z_{t+j})=0$ would produce an optimal rule for determining z_t (the Euler equation) of

$$(1+\phi+\beta\phi)z_t+\beta\phi E_t z_{t+1}-z_t^*=0 \tag{9.9}$$

and an ultimate solution for z_t of the form

$$z_t=\lambda z_{t-1}+\frac{\lambda}{(1-\phi)}E_t\sum_{j=0}^{\infty}(\beta\lambda)^j z_{t+j}^*, \tag{9.10}$$

where λ depends on β and ϕ. Thus z_t can be constructed by weighting together past and future expected values of z_t and z_t^*. Because expectations in 3G models were effectively of the perfect foresight variety, model-consistent expectations would mean that $E_t z_{t+j}^*=z_{t+j}^*$. But, in the practice, the expectations were taken to be modeled as a function of the steady-state model solution, a finite number of lagged values of z_t, and the solution for z_{t+j}^* from QPM itself. The weights attached to these components were prescribed by the modelers.

Nickell (1985) and Rotemberg (1982) noted that the quadratic optimization scheme described above would result in an ECM connecting z_t and z_t^*, when z_t^* was a scalar and followed an autoregressive process. Hence, effectively QPM was imposing a set of ECM equations that determined the outcomes for z_t by reference to the short-run steady-state values z_t^*.

As in 2G models nominal quantities were handled by making prices a markup on marginal costs and then structuring the relation to handle dynamics and expectations. As marginal costs were primarily wages, a Cobb–Douglas production function and perfect competition meant that the wage share in GDP (or real unit labor costs) was a constant in equilibrium. With these ideas, and expectations handled as described above, one might think of

the 3G Phillips curve as effectively having the form

$$\pi_t = \alpha_1 E_t \pi_{t-1} + (1 - \alpha_1) E_t \pi_{t+1} + \delta \Delta mc_t + \omega(p_{t-1} - mc_{t-1}), \qquad (9.11)$$

where mc_t was the log of nominal marginal cost and $mc_{t-1} - p_{t-1}$ was lagged real unit labor costs. Thus inflation was determined from past inflation, future expectations of inflation, current growth in nominal costs and the extent to which real unit labor costs were not constant.

9.4.2 Estimation and Evaluation

There was little formal estimation of the parameters of these models. Ratios such as consumption to income were often the main source of information used in setting values. When it was necessary to specify parameters determining dynamic responses there seems to have been significant interaction between modelers, policy advisers, and policy makers over whether the outcomes from the model with particular parameter values accorded with their views. Sometimes this involved studying the speed of adjustment after a shock while at other times estimates of quantities such as the sacrifice ratio would help in deciding on the balance between future and backward looking expectations (α_1 in the Phillips curve). Consequently, data did play some role in quantifying parameters, for example in QPM, but it was only used informally via the experience that had accumulated of the Canadian economy. Conceptually, one might think of this process as involving the use of a criterion function to match data (generally filtered) with simulated output from the models. The criterion function could then also be used to discriminate between different sets of parameter values. The exception to this strategy was when standard estimation methods were applied to the ECMs used in quantifying the second stage dynamics.

Evaluation of these models was rarely done. Indeed there was even an hostility toward data (see Colletti et al. 1996, p. 14), where they say about modeling in the Bank of Canada:

> There had been a systematic tendency to overfitting equations and too little attention paid to capturing the underlying economics. It was concluded that the model should focus on capturing the fundamental economics necessary to describe how the macro economy functions and, in particular, how policy works, and that it should be calibrated to reflect staff judgement on appropriate properties rather than estimated by econometric techniques.

Leaving this debate aside, given the way the models were used it would have been very difficult to perform a satisfactory evaluation of them. The reason was the method of producing a series on the short-run steady-state path z_t^*. The description given above was in fact too simplified. An implication of that account was that the steady-state solutions for the logs of the endogenous variables would be constructed from the exogenous variables by using a set of weights that are functions of the model parameters and that the latter would be assumed to be invariant over time. Such a scenario would

generally imply constancy in a number of ratios. For example, the investment to capital stock ratio would be a constant since, in steady state, it equals a parameter — the depreciation rate of capital. But, after examining the data, it was evident that these ratios were rarely constant, and often wandered far away from any fixed point. So, although one did need to assume some fixed values for the steady-state ratios (equivalently the model parameters), it also became necessary to make some allowance for the substantial time variation seen in ratios over any given data period. Failure to do so would constitute a gross mismatch of the data and model predictions. Consequently, a two-part strategy evolved to deal with this problem. It firstly involved smoothing the observed ratios with some filter to produce an adjusted ratio that changed slowly. Secondly, this adjusted ratio was forced to converge to whatever long-run ratio was prespecified in the steady-state model. Essentially this strategy meant that the steady-state model parameters were allowed to vary smoothly over time with the restriction that they converged to a set of final steady-state choices.[3]

Cast in terms of our discussion above, the time variation in z_t^* comes not only from exogenous variables, transition paths, etc., but can also occur due to time-varying model parameters. Without this latter source of variation a comparison on how well z_t^* (the model SRSS) tracks z_t (the data) would seem a useful diagnostic for how well the two paths match, but, if one can vary the parameters of the model in a complex way so as to get a better fit to the data, such a comparative exercise becomes meaningless. Thus one cannot satisfactorily evaluate the success of the static model constructed in the first stage of a 3G modeling exercise.[4]

Turning to the second stage of 3G model construction, if a series on z_t^* was available one might think about checking the dynamic representation chosen in that stage. But here resort was often made to polynomial adjustment schemes that introduced much higher order lags than the first order of the stylized ECM connecting z_t and z_t^* described above. In doing that one could almost be certain of getting a good fit to any historical series on z_t. For 3G models therefore the only satisfactory evaluation method probably resided in whether their clients were happy with the information provided.

In the description above attention was centered upon the "gap" between z_t and z_t^* and it therefore became natural to convert all the variables in the model to "gap" format, particularly when the model was used in forecasting mode. This enabled one to improve forecasting performance by augmenting the equations for z_t with variables that were zero in the steady state. Hence,

[3] Some parameters were held constant since not all ratios exhibited a substantial degree of time variation.

[4] In more technical terms, if there is such great variation in the time history of the ratio that it needs to be described as an $I(1)$ process, then the methods used were essentially eliminating a unit root in the observed ratio through a filtering operation such as the Hodrick–Prescott filter. Of course if the model predicts that the ratio is $I(0)$, and the data that it is $I(1)$, it might be thought that some modification of the steady-state model is needed. Simply ignoring the mismatch by eliminating the $I(1)$ behavior via filtering seems unsatisfactory.

in the case where z_t was the log of the price level, one could add on an output gap to the equation that came from the second stage optimization. Over time this emphasis on "gaps" gave rise to the miniature models known as New Keynesian, and today these small models are often used for policy analysis and some forecasting, e.g., Berg, Karam, and Laxton (2006). In some ways the philosophy underlying 3G models had much in common with that stream of computable general equilibrium (CGE) modeling stemming from Johansen (1960). In that literature models were log-linearized around some "steady-state" values and the computation of these steady states (often termed the benchmark data set) involved substantial manipulation of data on input-output tables, etc. Of course the CGE models were not in "real time" and so transition paths were essentially irrelevant. It was simply assumed that enough time had elapsed for a new steady state to be attained once a policy change was made.

Another feature of 3G models was that shocks became the focus of attention. In the academic literature shocks had become a dominant feature of models and, with the advent of policy rules, one could no longer think about changing variables such as government expenditure or the money supply, since these were now endogenous variables. Only exogenous shocks to them might be varied. However, although the language was stochastic, often the solution methods were essentially deterministic, and so there was no "clean" incorporation of shocks into the models.

An issue that arose when these models were applied to a small-open economy with the rest of the world being treated as exogenous was what modification needed to be made to ensure that agents did not borrow indefinitely at the fixed external rate of interest; see Schmidt-Grohe and Uribe (2003) for a discussion of strategies for dealing with this issue. In practice, two of these adjustments tended to be used to design models that ruled out such behavior. In the first, the infinitely lived consumer of the Ramsey model was replaced by agents with finite lives. This formulation could be shown to be equivalent to a model with a representative consumer whose discount rate depended on the probability of death as in Blanchard (1985) and Yaari (1965). A second approach was to have the risk premium attached to foreign debt rising with the level of foreign borrowing, so that eventually agents would not wish to borrow from foreign sources to finance consumption. The ratio of foreign debt to GDP therefore became a crucial element in the latter models and decision rules had to be constructed to ensure that this prescribed ratio was achieved in steady state.

9.5 Fourth Generation (4G) Models

A fourth generation of models has arisen in the early 2000s. Representatives are TOTEM (Bank of Canada, Murchinson and Rennison 2006); MAS (the Modelling and Simulation model of the Bank of Chile, Medina and

Soto 2006); GEM (the Global Economic Model of the IMF, Laxton and Pesenti 2003); BEQM (Bank of England Quarterly Model, Harrison et al. 2005); NEMO (Norwegian Economic Model at the Bank of Norway (Brubakk et al. 2006); The New Area Wide Model at the European Central Bank, Kai, Coenen, and Warne 2008); the RAMSES model at the Riksbank (Adolfson et al. 2007); AINO at the Bank of Finland (Kilponnen and Ripatti 2005); SIGMA (Erceg, Guerrieri, and Gust 2006) at the U.S. Federal Reserve; and KITT (Kiwi inflation targeting technology) at the Reserve Bank of New Zealand (Beneš et al. 2009).

9.5.1 Extensions of 3G Model Features

In some ways these new models represent a completion of the program for adapting the Ramsey model for macroeconometric use. As with 3G models they are designed to have an underlying steady-state representation. But other features of their design are different to what was standard with 3G models. Four of these are of particular importance.

Firstly, shocks are now becoming explicitly part of the model rather than being appended at the end of the modeling process. A shock is what remains unpredictable relative to an information set specified within the model, and so it is necessary to be explicit about what this information is. In addition, how persistent the shocks are becomes important to describing the complete dynamics of the model, and this makes it necessary to decide on the degree of persistence. Given that shocks are unobservable (they are essentially defined by the model itself) this inevitably points to the need to quantify the parameters of the model from data.

Secondly, there is now no second-stage process to introduce dynamics. Instead, the adjustment cost terms used to rationalize slow adjustment in 3G models now appear directly in the primary objective functions that lead to the agent's decision rules, i.e., the short- and long-run responses are found simultaneously rather than sequentially. Of course the logic of the two-stage process used in 3G models was a recognition that adjustment costs (and the parameters associated with them) do not affect the steady-state solutions, and it was only the transition paths between steady states that depended on those parameters. In fact, recognition of this feature was the motivation for adapting 3G models to an existing forecasting environment by treating the construction of dynamics in two steps.

Thirdly, the structural equations of the model are now kept in Euler equation form rather than using a partially solved out version as was characteristic of 3G models. Thus the optimal intertemporal rule describing consumption decisions appears in most 4G models as

$$C_t = \beta E_t(C_{t+1} R_{t+1}), \tag{9.12}$$

which contrasts with the 3G model approach that combines this relation with the wealth accumulation identity to express consumption as a function of financial wealth and labor income. One reason for doing so is that it is easier

to modify the model design through its Euler equations. An example is the extra dynamics introduced into consumption decisions by the use of habit persistence. This can take a number of forms, but often results in the addition of C_{t-1} to the equation to give

$$C_t = \beta E_t\left(C_{t-1}^h C_{t+1}^{1-h} R_{t+1}\right). \tag{9.13}$$

Finally, because shocks were an integral part of some of these models, solution methods needed to be shaped to account for them. Indeed, with this focus on shocks one had to be careful when referring to "forward" and "backward" expectations; all expectations are now formed using information available at time t, and so technically all depend on past observations (unless there are exogenous variables in the system). Thus the important feature becomes the relative weights to be attached to the available information at time t when forming expectations at different periods. A second consequence of the shift to a "shocks" perspective is that the distinction between "parameters" and "shocks" becomes blurry. Thus a depreciation rate might now be regarded as a random variable that evolves stochastically over time with an expected value equal to whatever specified value for it appears in the steady-state model. Thus this provides a formal way of allowing the model parameters to change, something that was only done in an *ad hoc* way in 3G models.

9.5.2 New Features of 4G Models

The modifications above are essentially adjustments to the basic strategies employed in the design of 3G models and are intended to produce a more precise and satisfactory statement of the design criteria. But there are also additions. Four can be mentioned.

1. Although the models are ultimately about aggregates the theoretical structure is now often based on studying the actions of heterogeneous units and providing an account of how these are to be aggregated. This heterogeneity is used in many contexts. Thus analysis often begins with different types of labor services, many intermediate goods being produced and used to make a final good, many types of imported goods, firms being differentiated in their price setting policies, etc. The question is then how one performs an aggregation of the micro decisions. The solution is an extensive use of methods popular in CGE modeling. These involve the presence of an "aggregator." This intermediary uses CES functions as a way of combining together the many separate items into a composite commodity. Thus aggregate output in a two sector model, Y_t, would be the following combination of the sectoral outputs Y_{it}

$$Y_t = \left[Y_{1t}^{-\rho} + Y_{2t}^{-\rho}\right]^{-1/\rho}, \quad \rho = \frac{1-\lambda}{\lambda}. \tag{9.14}$$

A continuum of micro-units over (0, 1) is generally used in place of a finite number as above, and, in such a case, Y_t would be represented as

$$Y_t = \left[\int_0^1 Y_{it}^{\frac{\lambda-1}{\lambda}} di\right]^{\frac{\lambda}{\lambda-1}}. \tag{9.15}$$

Profit maximization by the sectoral producers means that the amount of the sectoral output produced would depend on Y_t and the relative price $\frac{P_{it}}{P_t}$, with the functional form being

$$Y_{it} = Y_t \left(\frac{P_{it}}{P_t}\right)^{-\lambda}. \tag{9.16}$$

As well, the aggregate price level relates to the sectoral ones as

$$P_t^{1-\lambda} = \int_0^1 (P_{it})^{1-\lambda} di. \tag{9.17}$$

Models are then built for P_{it} and Y_{it} and aggregated with these functions. The method is well known from Dixit and Stiglitz (1977). Because of the use of CES functions any underlying heterogeneity has an impact only through the presence of parameters that describe the nature of the heterogeneity, i.e., the distribution of the micro decisions (say on P_{it}). Basing the model design on a microeconomic structure can potentially expand the range of information available for parameter estimation through the use of studies of microeconomic decision making.

2. In the case of firms following different pricing strategies the aggregation scheme just described forms the basis of the Calvo pricing model. In this some firms can optimally reset their prices each period and others need to follow a simple rule of thumb. Consequently, the heterogeneity in decisions about P_{it} can be summarized by a single parameter — the fraction of firms (ξ) who are able to optimally adjust their price at each point in time. The aggregate Phillips curve can then be shown to have the form

$$\pi_t - \overline{\pi} = \frac{1}{1+\beta}(\pi_{t-1} - \overline{\pi}) + \frac{(1-\xi)(1-\beta\xi)}{\xi(1+\beta)}(\mathrm{rmc}_t - \overline{\mathrm{rmc}}) + \frac{\beta}{1+\beta} E_t(\pi_{t+1} - \overline{\pi}) + \varepsilon_t, \tag{9.18}$$

where rmc_t is real marginal cost (or unit labor costs with a Cobb–Douglas production function) and $\overline{\pi}$ is a target rate of inflation. An appealing argument for building the curve up from a micro-unit level was that it allowed for monopolistic and monopsonistic behavior at that level rather than the competitive markets of the 3G models. Thus the rather awkward assumption used in QPM that there was a mark-up of prices over marginal costs in the short run, but that it went to zero in steady state (owing to the competitive markets assumption), can be dispelled.

It should be observed though that, although widespread, it is not always the case that the Calvo pricing structure is used in 4G models. Sometimes the approach used by Rotemberg (1982) is adopted. But the nature of the resulting Phillips curve is similar.

3. The steady state used in 3G models saw real variables such as output, capital, etc. as either a constant or following a deterministic growth path. This reflected the fact that labor augmenting technical change was taken as growing at a constant rate, basically following Solow (1956) and Swan (1956). Although initially in 4G models technology was treated as stationary, many models now allow the technical change to have a stochastic permanent component as well as a deterministic one. Thus the "steady-state" solution evolves stochastically over time. With some variables now having permanent components questions arise over how one should treat this fact when operationalizing the model, and we return to that later in the section.
4. Now that the models are treated as stochastically focused, when log-linearized they can be represented as structural equations of the form[5]

$$B_0 z_t^M = B_1 z_{t-1}^M + C E_t z_{t+1}^M + F \varepsilon_t, \tag{9.19}$$

where z_t^M and ε_t are the model variables and shocks, respectively. The solution to Equation 9.19 when ε_t has no serial correlation is[6]

$$z_t^M = A z_{t-1}^M + G \varepsilon_t. \tag{9.20}$$

Because it is possible that some of the model variables are not observed, it is useful to connect those variables that are observable, z_t^D, to the model variables via an observation equation

$$z_t^D = H z_t^M + \eta_t, \tag{9.21}$$

where η_t is what needs to be added on to the model solution to replicate the data. Here η_t will be termed the "tracking shocks." Altug (1989) pioneered this approach assuming that the η_t were i.i.d. and uncorrelated with model shocks. Ireland (2004) has a generalization of this where η_t can be serially correlated. Sometimes the η_t are referred to as "errors in variables," but many of the variables modeled, such as interest rates and exchange rates, are very accurately measured, and any mismatch is due to difficulties with the model rather than measurement issues. Equations 9.20 and 9.21 constitute a State Space Form (SSF) and is pivotal to estimation methods for those models in which not all model variables are observable, i.e., when the dimension of z_t^D is less than z_t^M.

[5] Of course the system may have higher order lags. Any exogenous variables are placed in z_t and assumed to evolve as a VAR.

[6] If the shocks ε_t follow a VAR(1) process then the solution to the system is a VAR(2), as shown in Kapetanios, Pagan, and Scott (2007). Note that, while A is a function solely of B_0, B_1, C, G will depend on these parameters plus any parameters describing the persistence in the shocks ε_t; see Binder and Pesaran (1995). This demarcation can be a very useful result.

9.5.3 Quantifying the Parameters of 4G Models

There is no one method of estimating the parameters that appear in 4G models. In some cases the approach used is the same as in 3G models. Broadly, this involved first estimating any parameters that appear in the steady state with observable ratios of variables, i.e., a method of moments estimator was implicitly being utilized. Secondly, parameters associated with the transitional paths were generally quantified by utilizing opinions about desirable model performance. Increasingly the latter strategy has been replaced by variants of maximum likelihood estimation.

9.5.3.1 Identification of the Parameters

The equations to be estimated are in Equation 9.19. A first complication in estimating this system comes from the presence of $E_t z_{t+1}$.[7] Now it is clear from Equation 9.20 that $E_t z_{t+1} = Az_t$ and so Equation 9.19 becomes

$$\Phi z_t = B_1 z_{t-1} + G\varepsilon_t, \tag{9.22}$$

where $\Phi = (B_0 - CA)$, which is a standard set of simultaneous equations. One could therefore ask whether Φ, B_1, and G are identifiable. But, since it is the parameters θ that appear in the 4G model which are ultimately of interest, i.e., those in B_0, B_1, and C, looking at identification of Φ, B_1, and G would just be a stepping stone toward examining whether θ is identified. In both instances one has to distinguish between whether there are different values of the parameters in a *given model* which would reproduce the second moments of the z_t (assuming it is stationary) and whether there is just *one model* that is consistent with those second moments. These are different questions. As Preston (1978) emphasized, the first is a question of *structural identification*, and so the conditions are effectively those of the Cowles Commission, as generalized by Rothenberg (1971). In contrast, the second depends upon what transformations are allowed in forming new models. If one can reallocate the dynamics across the equations of a given model to form a new model then they are like those in Hannan (1971). Even if the existing dynamics are to be retained, i.e., B_0, B_1, and C are fixed it may still be possible to recombine the shocks ε_t to $\zeta_t = U\varepsilon_t$, where U is nonsingular, so that G in Equation 9.22 becomes GU^{-1}. This results in a different set of impulse responses to the new shocks ζ_t, even though the second moments for z_t are identical to the model with the ε_t shocks. Such a recombination strategy is employed in the VAR sign restrictions literature to give new shocks which obey certain restrictions — in particular U is chosen there so that the shocks remain mutually uncorrelated. Now the new shocks essentially mean a new model has been found but it is one that is observationally equivalent to the old one (since the second moments of z_t are the same). This distinction between these two identification ideas is still not well understood. Many demonstrations of identification problems,

[7] We will ignore the distinction between observable and unobservable variables and drop the "M" for the moment.

such as Canova and Sala (2009), are concerned with structural identification, but recent work by Komunjer and Ng (2009) has been more about model identification. Whether there is a unique model might be of interest but, for estimation purposes, it is structural identification that is paramount.

In most situations B_0, B_1, and C are relatively sparse and so standard simultaneous equation identification conditions can be applied to identify Φ, B_1, and G, since there will be enough instruments to apply to each of the structural equations. Of course it may be that the instruments are weak and, in a finite sample, there is effectively a lack of identification. Indeed, many of the examples of identification difficulties that pertain to the structural equations of 4G models, such as the New Keynesian Phillips curve, do seem to be concerned with the presence of weak instruments – Mavroeidis (2004), and Nason and Smith (2005).

Often it is useful to ask whether Φ,B_1, and G can be identified before proceeding to query the identification status of θ. Since these parameters determine the impulse responses to shocks that might be sufficient for much policy analysis. However, it may be that, even when Φ and B_1 are identified, the mapping between these and the 4G model parameters, θ, might not be one to one, i.e., θ is not identified. If policy experiments involved changing the steady-state solutions then we will mostly want to identify θ rather than the impulse responses. Some of the experiments done to look at identification failures are examples of not being able to uniquely recover θ from $\{\Phi,B_1,G\}$. Generally, therefore one can think that there are two aspects to identification. One involves the ability to identify $\{\Phi,B_1,G\}$ from the data and the other is whether the model allows one to recover θ from these matrices. This distinction has been promoted in Iskrev (2007, 2009).

It should be noted that structural identification of 4G models is largely based on exclusion restrictions as proposed by the Cowles Commission. In most instances these models are therefore strongly overidentified. Even if they are not, there is a separate set of exclusion restrictions that need to be taken into account, namely, those that come from the standard assumption in these models that the shocks ε_t are contemporaneously uncorrelated. Those restrictions produce extra instruments that can be used for estimation that were not present in the analysis provided by the Cowles researchers, since they took the errors in their structural equations to be correlated.

9.5.3.2 Maximum Likelihood and Bayesian Estimation of Parameters

In studying identification issues A may be taken as known but, in estimation, a decision has to be made whether it should be found from a regression on Equation 9.20 ($\hat{A}$) or forced to be consistent with the 4G model, in which case A depends on values of the structural parameters θ. In the former case one can utilize limited information methods of estimation, allowing each structural equation to be estimated separately. For the latter a complete systems estimator is needed. Which one is to be used depends on the degree of robustness for the parameter estimates that one wants. Using Equation 9.20 to form an estimate of A (and hence measuring expectations) will be much more robust

to system mis-specification, i.e., $\hat{A}$ will be a consistent estimator of A provided the system generating the data can be represented by a VAR of the selected order, and it does not depend upon knowing the structural specification that generated the data. However, a more efficient estimator of A is available by utilizing the mapping between A and θ. As has been known for a long time, such efficiency can come at the expense of bias and inconsistency of estimators, unless the complete system is an adequate representation of the data. As Johansen (2005) has pointed out, this is a price of MLE, and it should not be assumed that the 4G model has the property of being a correct specification.

Making A depend upon θ has led to full information (FI) maximum likelihood (FIML) becoming a standard way of estimating smaller 4G models (those that are generally referred to as DSGE models). This contrasts with the earlier generations of models where limited information (LI) estimation methods prevailed, i.e., the equations (or subsets of them) were estimated separately, and the influence of the complete model was minimal. It is interesting to note that the wheel has almost come full circle as the recommendation by the Cowles Commission was to use FIML, but they were frustrated by the fact that computers were not powerful enough at that time for such an estimator to be effectively employed.

In practice the FIML estimator has increasingly been replaced by a Bayesian full information (BFI) estimator. In this estimates of θ comparable to FIML can be found by maximizing a criterion function $L(\theta) + \ln p(\theta)$, where $p(\theta)$ is the prior on θ and $L(\theta)$ is the log-likelihood. The resulting estimate of θ is often referred to as the mode of the posterior. It is clear that the FIML and the Bayesian FI mode (BFI) will converge as the sample size grows and the prior information becomes dominated. Hence any difficulties arising with FIML involving misspecification of the system cannot be avoided by using a Bayesian estimator. This seems to be widely misunderstood as one often sees comments that Bayesian methods do not require correct specification of the model.

An advantage of the Bayesian method is that there is often information about the range of possible values for θ, either from constraints such as the need to have a steady-state or from past knowledge that has accumulated among researchers. Imposing this information upon the MLE is rarely easy. It can be done by penalty functions, but often these make estimation quite difficult. Adding on $\ln p(\theta)$ to the log-likelihood generally means that the function being maximized is quite smooth in θ, and so estimation becomes much easier. We think that this advantage has been borne out in practice; the number of parameters being estimated in 4G models, like that of Smets and Wouters (2003) and the new area wide model, is quite large, and one suspects that ML estimation would be quite difficult. There is, however, a cost to Bayesian methods. Although sometimes it is portrayed as a way of "filling in the potholes" of the likelihood surface — for example, in Fernández-Villaverde (2009) — often it is more like a "highway redesign." Unlike penalty functions the use of a prior can severely change the shape of the function being optimized. In particular, if $L(\theta)$ is flat in θ then the choice of prior will become

very important in determining the estimated parameter values, and so one needs to have methods for detecting that.

To illustrate that Bayesian methods can easily hide the fact that the data has little to say about the estimation of certain parameters, take the exchange rate equation in the model of Lubik and Schorfheide (2007)

$$\Delta e_t - \pi_t = -(1-\alpha)\Delta q_t - \pi_t^*, \tag{9.23}$$

where e_t is the log of the exchange rate, q_t is the observable (exogenous) terms of trade and π_t^* is the (unobservable) foreign inflation rate. The latter are assumed to be generated as AR(1) processes with parameters ρ_q and ρ_{π^*}, respectively, and uncorrelated shocks (as befits the exogeneity assumption in force for Δq_t). Under these assumptions Equation 9.23 is actually a regression equation, with $\Delta e_t - \pi_t$ as dependent variable, Δq_t as the regressor and with first order serially correlated errors. Hence the FIML and LIML estimators should be close. However, there will be a difference between a Bayesian estimator based on limited and full information when there are informative priors about the other parameters of the system.

Table 9.1 gives the LIML estimates of the parameters of Equation 9.23 using UK data from Lubik and Schorfheide. Also reported are the BFI estimator, which estimates the complete system (this involves imposing a zero correlation between all shocks of the system), and a LI Bayesian (BLI) estimator that imposes only a zero correlation between Δq_t and π_t^*. Two BLI estimators are given depending on whether the prior is assumed to be Beta(a_1, a_2) or $N(a_1, a_2)$. The BFI estimator is performed with Beta priors (for parameters appearing in the remainder of the system priors are those in LubiK and Schorfheide). For the estimation of α the Beta prior has $a_1 = .2, a_2 = .05$, while for $\rho_{\pi^*}, a_1 = .8, a_2 = .5$. The normal priors set $\{a_1 = 0, a_2 = .05\}$ and $\{a_1 = .8, a_2 = .5\}$, respectively.

Now it is clear how important the prior is in changing the results. The β prior used for ρ_{π^*} is fairly close to what is traditionally used in estimating 4G models. With just the BFI results you would never discover that the value most consistent with the data is negative. As the BLI estimates show this is not a question of using a more efficient estimator. To get the Bayesian estimator to reveal the lack of information about α in the data it is necessary to choose the prior so as to encompass a wide range of values for the parameter, but often one sees a very restricted parameter range specified so as to get "sensible values,"mostly ruling out certain signs for the estimates. However, a "wrong

TABLE 9.1

FIML and Bayesian Estimates of the Parameters of Equation 9.23

	α		ρ_{π^*}	
	Mean Est.	**95% Range**	**Mean Est.**	**95% Range**
FIML/LIML	−0.11	−0.56–0.34	0.07	−0.13–0.32
BFI-Beta	0.19	0.12–0.27	0.39	0.39–0.67
BLI-Beta	0.19	0.06–0.31	0.44	0.29–0.59
BLI-normal	0.01	−0.07–0.08	0.08	−0.15–0.31

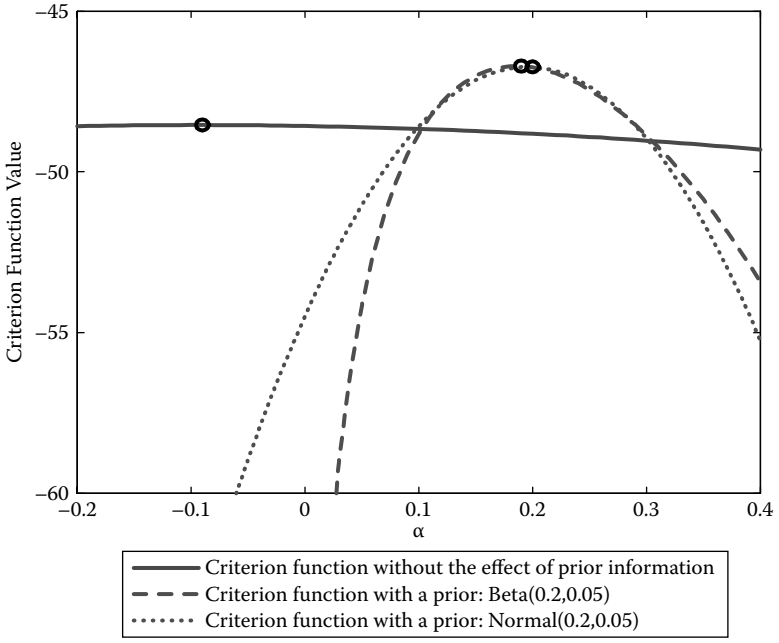

FIGURE 9.1
Log-likelihood and Bayesian mode criterion for α.

sign" can be very informative. In times past it was often taken to suggest that there are specification problems with the equation. In the case of α, a negative value is certainly unattractive, since it is meant to be an import share, but the proper way to interpret the MLE estimate is really that one can not estimate the parameter with any precision, rather than it is negative. What is disturbing about this example is that one does not get any such feeling from the Bayesian estimates, unless one allows for the possibility that the coefficient can easily be negative, as with the last prior. A different way of seeing how the prior has reshaped the surface is in Figure 9.1, which shows how the log-likelihood and the criterion generating the Bayesian modal estimate change with two priors. Notice how the prior can lead to the conclusion that this is a parameter whose value can be determined very precisely.

Although there is nothing surprising in these outcomes, the point is that the Bayesian estimates suggest the opposite, i.e., there seems to be a good deal of information in the sample, as shown by the fact that the mean of the prior for ρ_{π^*} is not contained in the 90% confidence interval for either of the Bayesian estimators. Thus a commonly suggested criterion that there are issues if the posterior and prior distributions coincide would not flag any warnings here. It leads one to ask why one would not just compare the Bayesian modal estimate and its implied ranges for the parameter value to those coming from the MLE as a check on undue influence from the prior? Oddly enough, this information is rarely supplied by those estimating 4G models with Bayesian methods.

It has been observed above that not all the model variables may be observed. This has the effect of potentially making the solved solution *in the observed variables* a VARMA rather than a VAR process.[8] This has to be allowed for when forming the likelihood. It is here that expressing the model and observation information in a state space form is very useful, since the likelihood can be computed recursively (at least when the shocks are normal) using the information provided by the Kalman filter. Most computer programs estimating 4G models use this method, e.g., the DYNARE program. Assuming that the process is a VAR in the observables can lead to quite large biases in the estimates of impulse responses unless there are enough observations to estimate a high order VAR (as that can approximate a VARMA process). For example, Kapetanios, Pagan, and Scott (2007) found that, for a model which was a smaller version of the 4G model BEQM, one needed a VAR(50) to recover the true impulse responses. Otherwise the biases were large when the sample size was that commonly available, around 200 observations, and the VAR order was chosen with standard statistical order selection methods such as BIC and AIC. Of course a VAR(50) is not something that is estimable in sample sizes like 200.

But there are limits to this strategy. One cannot have too many unobserved variables. Strong assumptions may need to be made about variances in order to achieve identification of these parameters if there is a big discrepancy, something that does not seem to be appreciated by many of those applying the methods. For example, it is not enough to follow Smets and Wouters (2003, p. 1140) who say "Identification is achieved by assuming that four of the ten shocks follow a white noise process. This allows us to distinguish those shocks from the persistent 'technology and preference' shocks and the inflation objective shock."

To see the problem that arises with having an excess of unobservables consider the simplest case where there is one observed variable y_t but two unobserved components y_{1t} and y_{2t}. One of these components (y_{1t}) follows an AR(1) with parameter ρ_1 and innovation variance σ_1^2, and the other is white noise ($\rho_2 = 0$) with variance σ_2^2. Then we would have

$$(1 - \rho_1 L) y_t = (1 - \rho_1 L) y_{1t} + (1 - \rho_1 L) y_{2t}, \tag{9.24}$$

and it is clear that, as $\frac{\sigma_2^2}{\sigma_1^2}$ becomes large, it becomes impossible to identify ρ_1. In this case the likelihood is flat in ρ_1, and any prior placed on ρ_1 will effectively determine the value of ρ_1 that results. To avoid this situation a prior would need to be placed on the relative variance and not just the values of ρ_1 and ρ_2, as Smets and Wouters argue. To illustrate this we simulated some data from the setup above and then estimated ρ_1 with a Beta prior centered at

[8] There is a large literature on this and related issues now, e.g., Fernandez-Villaverde et al. (2007). Simple conditions under which this occurs are set out in Fukač and Pagan (2007). Thus in the basic Real Business Cycle model in King, Plosser, and Rebelo (1988), variables such as consumption can be eliminated and the model will remain a VAR, but the capital stock cannot be.

TABLE 9.2

An Example of a Too-Many-Unobservables Model Estimation (Estimates of ρ_1 and 90% Confidence Interval)

	True σ_2^2/σ_1^2		
Prior	**1**	**2**	**5**
$\rho_1 = 0.85$	0.67	0.71	0.80
	[0.49–0.84]	[0.53–0.89]	[0.64–0.94]
$\rho_1 = 0.50$	0.46	0.48	0.49
	[0.32–0.60]	[0.31–0.62]	[0.35–0.65]
$\rho_1 = 0.30$	0.28	0.28	0.29
	[0.12–0.41]	[0.13–0.46]	[0.12–0.44]

Note: We use a Beta prior on ρ_1, with a standard error 0.1. The true value is $\rho_1 = 0.3$. For σ_1 and σ_2 we use an inverse gamma with a mean 1 and standard error 4 as a prior.

different values. The true value of ρ_1 is .3 and Table 9.2 shows the posterior mode for different values of $\frac{\sigma_2^2}{\sigma_1^2}$. It is clear that recovering the true value of ρ_1 is extremely difficult if the type of prior used in many 4G models is adopted.

9.5.4 Handling Permanent Components

Increasingly it has been recognized that there are likely to be permanent components in the data and these must be introduced into the model design in some way. Most commonly this is done by making the log of the level of technology, A_t, an integrated process. Then to keep ratios such as the real capital-output and consumption-output constant in equilibrium, it follows that the permanent components of capital, output, and consumption, must be identical. Obviously the fact that ratios are to be taken as constant in equilibrium implies co-integration between the logs of the variables making up the ratio, and the co-integration vectors have the specific form of $(1 \quad -1)$. To see the implication of such co-integration assume that production is done via a Cobb–Douglas production function of the form $Y_t = K_t^{\alpha}(A_t H_t P_t)^{1-\alpha}$, where H_t is hours worked and P_t is the potential work force. In most models P_t is taken to grow exogenously and it is H_t that fluctuates with the latter being regarded as a stationary process with some average (steady-state) value of H^*. Potential output is then naturally defined as the permanent component of Y_t, Y_t^P. Under the restriction mentioned above that $Y_t^P = A_t^P = K_t^P$,

$$Y_t^P = (A_t^P)^{\alpha}(A_t^P H^* P_t)^{1-\alpha} = A_t^P (H^* P_t)^{1-\alpha}. \tag{9.25}$$

Taking logs and defining an output gap as the transitory component

$$\ln Y_t - \ln Y_t^P = \ln A_t - \ln A_t^P + (1-\alpha)\ln(H_t/H^*), \tag{9.26}$$

shows that the output gap depends upon the transitory component of technology, as well as the deviations of hours from its steady-state value H^*. In the special case when $\ln A_t$ is a pure random walk, the transitory component of $\ln A_t$ is zero. This special case is used quite extensively.

Models exist in the literature where there is more than one permanent component. The presence of more than one generally arises from noticing that the ratios of certain variables cannot be reasonably treated as a constant in the long run. In some instances this lack of stability is due to changes in relative prices. In these cases it is often the nominal rather than the real ratios that appear to be relatively constant, suggesting that the case be handled within a 4G model by employing a second unobservable permanent component that drives the relative prices. An example of a 4G model which incorporates such an adjustment is the KITT model (Beneš et al. 2009).

How does one handle permanent components in the solution and estimation of 4G models? Two strategies are available. One involves formulating the optimization problems used to get the Euler equations of the 4G models in such a way that any $I(1)$ variable appears as a ratio to its permanent component. In this variant, the utility function would be expressed in terms of $\frac{C_t}{C_t^P}$. An example of where this was done is Del Negro and Schorfheide (2008). The second strategy has been to reexpress the Euler equations derived from functions of the levels of the variables in terms of such ratios, e.g., the consumption Euler equation in Equation 9.13 would become

$$\frac{C_t}{C_t^P} = E_t\left[\left(\frac{C_{t-1}}{C_{t-1}^P}\right)^h \left(\frac{C_{t+1}}{C_{t+1}^P}\right)^{1-h} R_{t+1}\frac{(C_{t-1}^P)^h (C_{t+1}^P)^{(1-h)}}{C_t^P}\right]. \tag{9.27}$$

After log linearization this is

$$\zeta_t = h\zeta_{t-1} + E_t\left\{(1-h)\zeta_{t+1} + R_{t+1} + (1-h)\Delta c_{t+1}^P\right\} - h\Delta c_t^P, \tag{9.28}$$

where $\zeta_t = \ln C_t - \ln C_t^P$. An assumption now needs to be made concerning how $\Delta a_t = \Delta c_t^P$ is to be generated. In the special case where $\Delta a_t = \Delta \ln A_t = \varepsilon_t^a$ and ε_t^a is white noise, $E_t\Delta c_{t+1}^P = E_t\Delta a_{t+1}^P = E_t\varepsilon_{t+1}^a = 0$.

Which of these two strategies is best is a question that has not been examined much. Certainly they lead to different specifications for the Euler equations of any model. The presence of permanent components in technology makes it highly unlikely that any 4G model can be represented as a VAR and so estimation using the on-model approach is best done within the framework of an SSF. This simply involves specifying $\Delta\zeta_t = \Delta c_t - \Delta c_t^p$ as an observation equation, with Δc_t^P being latent and Δc_t being observed.

Notice that what the above strategy does is to replace any $I(1)$ series with their transitory components or "gaps." Essentially it is performing a multivariate Beveridge–Nelson decomposition of the $I(1)$ variables into their permanent and transitory components. However, often one sees a second strategy, which involves an "off-model" approach wherein permanent components are removed from variables by a filter that is not model consistent. By far the most popular would be the Hodrick–Prescott (HP) filter. Econometrically using off-model filters is a bad idea. To see this consider the consequences of working with HP filtered data. To assess these we note that the HP filter is a two-sided filter which, when applied to a variable y_t, produces a transitory

component of $\Sigma_{j=-T}^{j=T}\omega_j \Delta y_{t-j}$.[9] Now, if this component is used as a regressor, the fact that it involves Δy_{t+j} at time t means that one would get inconsistent estimators of the parameters attached to the gaps. Moreover, the correlation of the regressor with Δy_{t+j} is likely to contaminate estimators of other parameters. Even if one used a one-sided version of this filter it is well known – (see Harvey and Jaeger [1993] and Kaiser and Maravell [2002]) that the filter is designed to extract a permanent component from a series that is $I(2)$, not one that is $I(1)$, and hence it is not model consistent unless $\ln A_t$ is $I(2)$; see Fukač and Pagan (2010) for more details. Few 4G modelers are prepared to make that assumption.

9.5.5 Evaluation Issues

Evaluation really has two dimensions to it. One concentrates on the operating characteristics of the model and whether these are "sensible." The other is more about the ability of the model to match the data along a variety of dimensions. The two themes are not really independent but it is useful to make the distinction. Thus it might be that while a model could produce reasonable impulse responses, it may not produce a close match to the data, and conversely.

9.5.5.1 Operating Characteristics

Standard questions that are often asked about the operating characteristics of the model are whether the impulse responses to selected shocks are reasonable and what the relative importance of various shocks are to the explanation of (say) output growth. Although the latter is often answered by recourse to variance decompositions, perhaps a better question to ask is how important the assumptions made about the dynamics of shocks are to the solutions, as it seems crucial to know how much of the operating characteristics and fit to data comes from the economics and how much from exogenous assumptions. This concern stems back at least to Cogley and Nason (1993) who argued that standard RBC models produced weak dynamics if shocks were not highly serially correlated. It would seem important that one investigate this question by examining the impact of setting the serial correlation in the shocks to zero.

The appropriate strategy for assessing operating characteristics depends on whether the model parameters have been formally or informally quantified. If done informally researchers such as Amano et al. (2002) and Canova (1994) have asked the question of whether there is a set of such parameters that would be capable of generating some of the outcomes seen in the data, e.g.,

[9] Simulating data from y_t when it is a pure random walk, and then regressing the measured HP transitory component ($\lambda = 1600$) on to $\Delta y_{t\pm j}$, $j = 0, \ldots, 10$, gives an R^2 of .98 and $\omega_0 = .47$, ω_j ($j = 1., , 10$) = {.42, .37, .32, .27, .23, .18, .15, .15, .11, .09, .06}. It is also the case that $\omega_j = w_{|j|-1}$, $j = -1, \ldots, -10$. When future values of Δy_t were dropped from the regression the R^2 dropped to .5, emphasising the importance of future values of Δy_t in the determination of the HP transitory component.

ratios ϕ such as (say) the consumption-income ratio. This ratio is a function of the model parameters θ. The existing value used for θ in the model, θ^*, is then taken as one element in a set and a search is conducted over the set to see what sort of variation would occur in the resulting value of ϕ. If it is hard to reproduce the observed value of ϕ in the data, $\hat{\phi}$, then the model might be regarded as suspect. In this approach possible values of model parameters are selected to trace out a range of values of ϕ. An efficient way of doing this search is a pseudo-Bayesian one in which trial values of θ are selected from a multivariate density constructed to be consistent with the potential range of values of θ. This enables the corresponding density for ϕ to be determined. If the observed value $\hat{\phi}$ lies too far in the tails of the resulting density of ϕ, one would regard the model as inadequately explaining whatever feature is summarized by ϕ. A second approach treats the parameter values entered into the model, θ^*, as constant and asks whether the estimate $\hat{\phi}$ is close to the value $\phi^* = \phi(\theta^*)$ implied by the model. This is simply an encompassing test of the hypothesis that $\phi = \phi^*$.

9.5.5.2 Matching Data

Since 4G models are structural models there are many tests that could be carried out regarding their implied co-integrating and co-trending vectors, adequacy of the individual equations, etc. Moreover many of the old techniques used in 1G and 2G models, such as an examination of the tracking performance of the model, might be applied. But there are some issues which are specific to 4G models that need to be addressed in designing such tests.

In the first and second generation of models a primary way of assessing their quality was via historical simulation of them under a given path for any exogenous variables. It would seem important that we see such model tracking exercises for 4G models, as the plots of the paths are often very revealing about model performance, far more than might be gleaned from any examination of just a few serial correlation coefficients and bivariate correlations, which has been the standard way of looking at 4G model output to date. It is not that one should avoid computing moments for comparison, but it seems to have been overdone in comparison to tests that focus more on the uses of these models such as forecasting (which is effectively what the tracking exercise is about).

Now a problem arises in doing such exercises for 4G models. If the model's shocks are taken to be an integral part of it then there is no way to assess the model's tracking ability, since the shocks always adjust to produce a perfect match to the data. Put another way, there is no such thing as a residual in 4G models. The only exception to that is when we explicitly allow for tracking shocks, as described earlier, and this technology has sometimes been used to examine the fit. The main difficulty in doing so is the assumption used in setting up the SSF that the tracking shocks and model shocks are uncorrelated (since one cannot generally estimate such a parameter from the likelihood). Some relaxation of this assumption is needed, i.e., an auxiliary criterion needs to be supplied that can be used to set a value for the correlation. Watson (1993)

suggested that one find the correlation that minimized the gap between the spectra of the model and the data, as that produces the tracking outcome most favorable to the 4G model. Oddly enough Watson's approach does not seem to have been used much, although it is obviously a very appealing way of getting some feel for how well a 4G model is performing.

Rather than focus on tracking one might ask whether the dynamics are adequately captured by the model. One way to examine this is to compare the VAR implied by the model with that in the data. Canova, Finn, and Pagan (1994) proposed this. In small models this seems to be a reasonable idea but, in large models, it is unlikely to be very useful, as there are just too many coefficients to fit in the VAR. Consequently, the test is likely to lack power. Focusing on a subset of the VAR coefficients might be instructive. Thus Fukač and Pagan (2010) suggest a comparison of $E_t z_{t+1}$ generated from the model with that from a VAR. As there are only a few expectations in most 4G models this is likely to result in a more powerful test and has the added advantage of possessing some economic meaning. They found that the inflation expectations generated by the Lubik and Schorfheide (2007) model failed to match those from a VAR fitted to UK data.

A different way of performing "parameter reduction" that has become popular is due to Del Negro et al. (2007) — the so-called DSGE-VAR approach. To explain this in a simple way consider the AR(1) equation

$$z_t = \rho z_{t-1} + e_t, \tag{9.29}$$

where e_t has variance of unity. Now suppose that a 4G model implies that $\rho = \rho_0$, and that the variance of the shock is correctly maintained to be unity. Then we might think about estimating ρ using a prior $N(\rho_0, \frac{1}{\lambda T})$, where T is the sample size. As λ increases we will end up with the prior concentrating upon ρ_0 while, as it tends to zero, the prior becomes very diffuse. In terms of the criterion used to get a Bayesian modal estimate this would mean that the likelihood will be a function of ρ but the other component of the criterion — the log of the prior — would depend on λ. Hence we could choose different λ and see which produces the highest value of the criterion (or even the highest value of the density of z_t when ρ is replaced by its various model estimates as λ varies). For a scalar case this is not very interesting as we would presumably choose the λ that reproduces the OLS estimate of ρ (at least in large samples) but in a multivariate case this is not so. Basically the method works by reducing the VAR parameters down to a scalar measure, just as in computing expectations. As λ varies one is effectively conducting a sensitivity analysis.

9.6 Conclusion

The chapter has looked at the development of macroeconometric models over the past sixty years. In particular the models that have been used for analysing policy options. We argue that there have been four generations of

these. Each generation has evolved new features that have been partly drawn from the developing academic literature and partly from the perceived weaknesses in the previous generation. Overall, the evolution has been governed by a desire to answer a set of basic questions and sometimes by what can be achieved using new computational methods. We have spent a considerable amount of time on the final generation of models, exploring some of the problems that have arisen in how these models are implemented and quantified. It is unlikely that there will be just four generations of models. Those who work with them know that they constantly need to be thinking about the next generation in order to respond to developments in the macroeconomy, to new ideas about the interaction of agents within the economy, and to new data sources and methods of analyzing them.

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