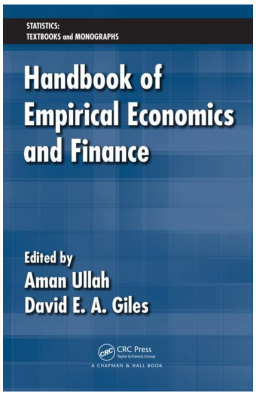

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The Unconventional Dynamics of Economic and Financial Aggregates

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The Unconventional Dynamics of Economic and Financial Aggregates

Karim M. Abadir and Gabriel Talmain

CONTENTS

8.1 Introduction

Time series models have provided econometricians with a rich toolbox from which to choose. Linear ARIMA models have been very influential and have enhanced our understanding of many empirical features of economics and finance. As with any scientific endeavor, data have emerged that show the need for refinements and improvements over existing models.

Nonlinear models have gained popularity in recent times, but which one do we choose from? Once we move away from linear models, there is a huge variety on offer. Surely, economic theory should provide the guiding light, insofar as economics and finance are the subject in question. Abadir and Talmain (2002) provided one possible answer. This chapter is mainly a summary of the econometric aspects of the line of research started by that paper.

The main result of that literature is that macroeconomic and aggregate financial series follow a nonlinear long-memory process that requires new econometric tools. It also shows that integrated series (which are a special case of the new process) are not the norm in our subject, and proposes a new approach to econometric modeling.

8.2 The Economic Origins of the Nonlinear Long-Memory

Abadir and Talmain (AT) started with a micro-founded macro model. It was a standard real business cycle (RBC) model, except that it allowed for heterogeneity: the "representative firm" assumption was dropped. They worked out the intertemporal general equilibrium solution for the economy, and the result was an explicit dynamic equation for GDP and all the variables that move along with it.

It was well known, long before AT, that heterogeneity and aggregation led to long-memory; e.g., see Robinson (1978) and Granger (1980) for a start of the literature on linear aggregation of ARIMA models, and Granger and Joyeux (1980) and Hosking (1981) for the introduction of long-memory models.[1] But in economics, there is an inherent nonlinearity which makes linear aggregation results incomplete. Let us illustrate the nonlinearity in the simplest possible aggregation context; see AT for the more general CES-type aggregation.

Decompose GDP, denoted by Y, into the outputs $Y(1), Y(2), \ldots$ of firms (alternatively, sectors) in the economy as

$$Y := Y(1) + Y(2) + \cdots = \mathrm{e}^{y(1)} + \mathrm{e}^{y(2)} + \cdots,$$

where we write the expression in terms of $y(i) := \log Y(i)$ ($i = 1, 2, \ldots$) to consider percentage changes in $Y(i)$ (and to make sure that models to be chosen for $y(i)$ keep $Y(i) > 0$, but this can be achieved by other methods too). With probability 1,

$$\mathrm{e}^{y(1)} + \mathrm{e}^{y(2)} + \cdots \neq \mathrm{e}^{y(1)+y(2)+\cdots},$$

where the right-hand side is what linear aggregation entails. The right-hand side is the aggregation considered in the literature, typically with $y(i) \sim \mathrm{ARIMA}(p_i, d_i, q_i)$, but it is not what is needed in macroeconomics. AT (especially p. 765) show that important features are missed by linearization when aggregating dynamic series.

One implication of the nonlinear aggregation is that the auto-correlation function (ACF) ρ_τ of the logarithm of GDP and other variables moving with it take the common form

$$\rho_\tau := \frac{\mathrm{cov}(y_t, y_{t-\tau})}{\sqrt{\mathrm{var}(y_t)\mathrm{var}(y_{t-\tau})}} = \frac{1 - a\,[1 - \cos(\omega\tau)]}{1 + b\tau^c}, \tag{8.1}$$

[1] A time series is said to have long memory if its autocorrelations dampen very slowly, more so than the exponential decay rate of stationary autoregressive models but faster than the permanent memory of unit roots. Unlike the latter, long-memory series revert to their (possibly trending) means.

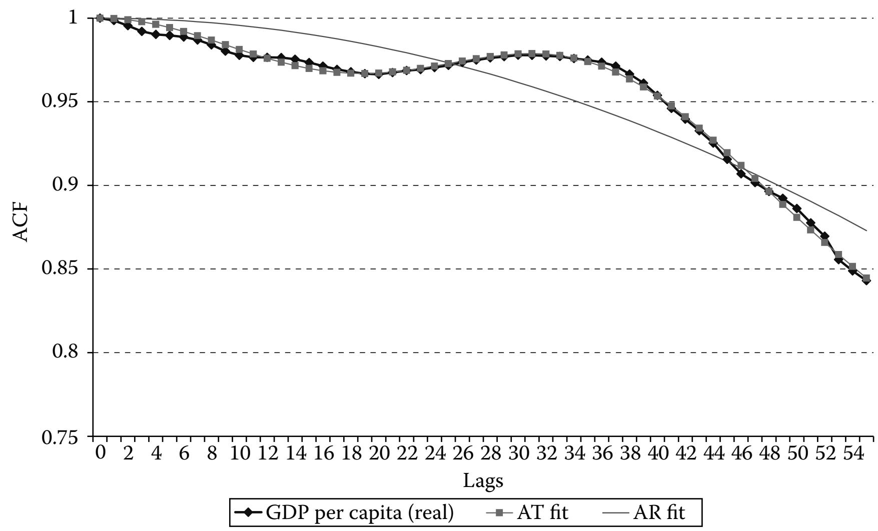

FIGURE 8.1
ACF of the log of U.S. real GDP per capita over 1929–2004.

where the subscript of y denotes the time period and a, b, c, ω depend on the parameters of the underlying economy but differ across variables.[2] Abadir, Caggiano, and Talmain (2006) tried this on all the available macroeconomic and aggregate financial data, about twice as many as (and including the ones) in Nelson and Plosser (1982). The result was an overwhelming rejection of AR-type models and the shape they imply for ACFs, as opposed to the one implied by Equation 8.1. For example, for the ACF of the log of U.S. real GDP per capita over 1929–2004, Figure 8.1 presents the fit of the best AR(p) model (it turns out that $p = 2$ with one root of almost 1) by the undecorated solid line, compared to the fit of Equation 8.1 by nonlinear LS. Linear models, like ARIMA, are simply incapable of allowing for sharp turning points that we can see in the decay of memory. The empirical ACFs found that there is typically an initial period where persistence is high, almost like a unit-root with a virtually flat ACF, then a sudden loss of memory. We can illustrate this also in the time domain in Figure 8.2, where we see that the log of real GDP per capita is evolving around a linear time trend, well within small variance bands that don't expand over time (unlike unit-root processes whose variance expands linearly to infinity as time passes).

ACFs of this shape have important implications for macroeconomic policymakers, as Abadir, Caggiano, and Talmain (2006) show. For example, if an economy is starting to slow down, such ACFs predict that it will produce a long sequence of small signs of a slowdown followed by an abrupt decline. When only the small signs have appeared, no-one fitting a linear (e.g., AR)

[2] The restrictions b, c, $\omega > 0$ apply, but the restriction on a cannot be expressed explicitly.

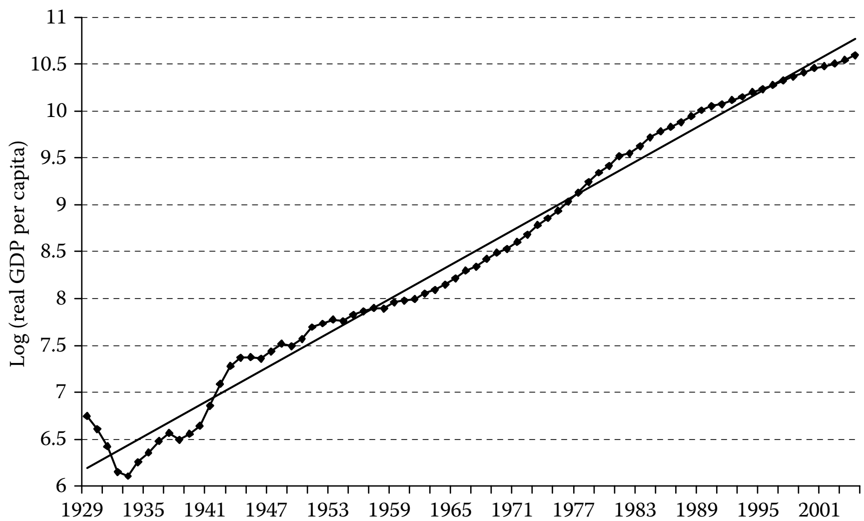

FIGURE 8.2
Time series of the log of U.S. real GDP per capita over 1929–2004.

model would be able to guess the substantial turning point that is about to occur. Another implication is that any stimulus that is applied to the economy should be timed to start well before the abrupt decline of the economy has taken place, and will take a long time to have an impact (and will eventually wear off unlike in unit root models). Consequently, a gradualist macroeconomic policy will not yield the desired results because it will be a case of too little and too late. In other words, a gradualist approach can be compatible with linear models but will be disastrous in the context of the ACFs that arise from macroeconomic data and that are compatible with the nonlinear dynamics generated by the general-equilibrium model of AT.

The ACF shape has important implications for econometric methods also. The long-memory cycles it generates require the consideration of singularities at frequencies other than 0 in spectral analysis. In fact, if a is close to 1 in the ACF (Equation 8.1), Fourier inversion produces a spectrum $f(\lambda)$ that is approximately proportional to $|\lambda - \omega|^{c-1}$; that is, at frequency ω, there is a singularity when $c \in (0, 1)$. For I(d) series having $d \in (0, \frac{1}{2})$, the spectrum has a singularity *at the origin* that is proportional to $|\lambda|^{-2d}$, giving the correspondence $c = 1 - 2d$ in the special case of $\omega = 0$. This correspondence holds also *in the tails* of the ACFs of the two processes when $\omega = 0$.

I(d) models are a special case of the new process. We therefore need to go beyond I(d) models and consider the estimation of spectral densities near singularities that are not necessarily located at the origin, as a counterpart (when $a \approx 1$) to the ACF-domain estimation mentioned earlier. Giraitis, Hidalgo, and Robinson (2001) and Hidalgo (2005) give a frequency-domain method of estimating ω and d, when $d \in (0, \frac{1}{2})$.

For $a \approx 1$, we introduce the following definition.

Definition 8.1 *A process is said to be of cyclical long-memory, respectively with parameters* $\omega \in [0, \pi]$ *and* $d \in (0, \frac{1}{2})$*, if it has a spectrum* $f(\lambda)$ *that is proportional to* $|\lambda - \omega|^{-2d}$ *as* $\lambda \to \omega$ *and is bounded elsewhere. Such a process is denoted by* $\mathrm{CM}(\omega, d)$*, with the special case* $\mathrm{CM}(0, d) = \mathrm{I}(d)$.

It is no wonder that a statistical model with cycles arises from a real business cycle model. Note that integrated processes cannot generate cycles that have long memory because their spectrum is bounded at $\omega \neq 0$. They can only generate short transient cycles that are not sufficiently long for macroeconomics.

When a is not close to 1 in the ACF (Equation 8.1), the result of the Fourier inversion is approximately a linear combination of one $\mathrm{I}(d)$ and one $\mathrm{CM}(\omega, d)$ when $\omega \neq 0$. Here, too, the approximation arises from the inversion focusing more on the tail of the ACF and neglecting to some extent the initial concave part of the ACF in Equation 8.1.

But if the individual series are not of the *integrated* type, can we talk of *co-integrated* series? It is an approximation that many not be adequate enough. What about the modification of co-integration modeling for variables that have this new type of dynamics? Abadir and Talmain (2008) propose a solution. We summarize it in the next section, and present an additional definition to complement Definition 8.1.

8.3 Modeling Co-Movements for Series with Nonlinear Long-Memory

This section contains three parts. We start with the specification, estimation, and inference in a model where the residual's dynamics are allowed to have the ACF in Equation 8.1. We then explore some empirical implications of such a model. Finally, we introduce a special case of the model that implies an extension of co-integration to allow for co-movements of CM processes.

8.3.1 Econometric Model

Suppose we have a sample of $t = 1, \ldots, T$ observations. To simplify the exposition, consider the model

$$z = X\beta + u, \tag{8.2}$$

where z is $T \times 1$ and β is $k \times 1$. The matrix X can contain lagged dependent variables, so that we cover autoregressive distributed-lag models (e.g., used in co-integration analysis) as one of the special cases. The vector u contains the residual dynamics of the adjustment of z toward its fundamental value $X\beta$. By definition, $\boldsymbol{u}$ is centered around zero and is mean-reverting, otherwise z will not revert to its fundamental value. We write $\boldsymbol{u} \sim \mathrm{D}(\mathbf{0}, \boldsymbol{\Sigma})$, where $\boldsymbol{\Sigma}$ is the $T \times T$ autocovariance matrix of the u's. The autocorrelation matrix of $\boldsymbol{u}$ is denoted by $\boldsymbol{R}$, and Abadir and Talmain (2008) use Equation 8.1 to parameterize the typical ijth element $\rho_{|i-j|}$ of $\boldsymbol{R}$. There are two implications to u_t being mean-reverting (which is a testable assumption). First, $\boldsymbol{\Sigma}$ is proportional to $\boldsymbol{R}$. Second, the ML estimator of β and the ACF parameters in $\boldsymbol{u}$ is consistent. The asymptotic distribution will depend on the properties of the variables, but if the estimated residuals are found to satisfy $c > \frac{1}{2}$ (implying square-summability of ρ_τ), then standard t, F, LR tests are justified asymptotically.[3] This condition on c is sufficient but not necessary, and we have found it to hold in practice when dealing with macro and financial series.

The quasi maximum likelihood (QML) procedure of Abadir and Talmain (2008) estimates jointly the parameters β and the ACF parameters in ρ_τ of $\boldsymbol{u}$. They remove the sample mean of each variable in Equation 8.2 to avoid multicollinearity in practice, with the constant term in $\boldsymbol{X}$ redefined accordingly. They also assume that $\boldsymbol{X}$ is weakly exogenous (see Engle, Hendry, and Richard 1983) for the parameters of Equation 8.2.

For any given $\boldsymbol{R}$, define

$$\widehat{\beta}_{\boldsymbol{R}} := \left(\boldsymbol{X}'\boldsymbol{R}^{-1}\boldsymbol{X}\right)^{-1} \boldsymbol{X}'\boldsymbol{R}^{-1}\boldsymbol{z} \tag{8.3}$$

as a function of $\boldsymbol{R}$. Denoting the determinant of a matrix $\boldsymbol{M}$ by $|\boldsymbol{M}|$, Abadir and Talmain (2008) show that the QML estimator (QMLE) of $\boldsymbol{R}$ is obtained by maximizing the concentrated log-likelihood

$$-\log \left| \left(\boldsymbol{z} - \boldsymbol{X}\widehat{\beta}_{\boldsymbol{R}}\right)' \boldsymbol{R}^{-1} \left(\boldsymbol{z} - \boldsymbol{X}\widehat{\beta}_{\boldsymbol{R}}\right) \boldsymbol{R} \right| \tag{8.4}$$

with respect to the parameters of the ACF: the optimization of the joint likelihood (for $\boldsymbol{\Sigma}$ and β) now depends on only four parameters that are given in Equation 8.1 and that determine the whole autocorrelation matrix $\boldsymbol{R}$. Once the optimal value $\widehat{\boldsymbol{R}}$ of $\boldsymbol{R}$ is obtained, the QMLE of β is $\widehat{\beta} \equiv \widehat{\beta}_{\widehat{\boldsymbol{R}}}$.

8.3.2 Empirical Implications

One is often interested in detecting the presence of co-movements between series. This may be for the purpose of empirically validating theoretical work, producing predictions, or determining optimal policies. In practice, one is often frustrated by the results produced by co-integration analysis. The theory of purchasing power parity (PPP) is typically tested using co-integration.

[3] This is a case where the results of Tsay and Chung (2000) on the divergent behavior of t-statistics do not apply, since the condition on c corresponds to the case of the series having $d < \frac{1}{4}$.

Generally, the findings are that PPP does not hold in the short run and deviations from PPP are cycling around the theoretical value at very low frequency, implying that the estimated reversion to PPP is, if at all, unrealistically slow.

Even when the series have less memory, dynamic modeling of comovements can spring surprises. According to the uncovered interest parity (UIP) theory, no contemporaneous variable should be able to predict the future excess returns in investing in a foreign asset. However, researchers have consistently found a strong negative relation between future excess returns and the forward premium on a currency. With the usual interpretation of the forward rate as a predictor of the future spot exchange rate, this would imply the irrational result that a currency is expected to depreciate in periods when assets denominated in this currency actually do produce systematic excess returns!

These "anomalies" or "paradoxes" are what one would find if the true nature of the relation between the variables is of the type in Equation 8.2, but the possibility of unconventional dynamics for $\boldsymbol{u}$ has been neglected. Co-integration would try to force a noncyclical zero-frequency pattern on this residual term which, in reality, is slowly cycling. By allowing for the possibility of long-memory cycles, the methodology described above brings to light the true nature of the residuals and, thus, of the true relation between the co-moving variables. The "long-run" relation between economic variables often involves long cycles of adjustment.

8.3.3 Special Case: co-CM

The model in Equation 8.2 avoids the question of the individual ω, d in each of the series contained in $\boldsymbol{z}, \boldsymbol{X}$. It just states that the dynamics of adjustment to the fundamental value (through changes in $\boldsymbol{u}$) is of the general AT type. A way in which this can arise is through the following special CM case of the AT process, where we use a bivariate context to simplify the illustration and to show how it generalizes the notion of co-integration.

Definition 8.2 *Two processes are said to be linearly co-CM if they are both* CM(ω, d) *and there exists a linear combination that is* CM(ω, s) *with* $s < d$.

This follows by the same spectral methods used in Granger (1981, Section 4). The definition can be extended to allow for nonlinear co-CM, for example, if $z_t = g(x_t) + u_t$ with g a nonlinear function. For the effect on the ACF (hence on ω, d) of parametric nonlinear transformations, see Abadir and Talmain (2005).

In Equation 8.2, it was not assumed that $s < d$. In fact, in the UIP application in Abadir and Talmain (2008), we had the ACF equivalent of $s = d$ because it was a trivial co-CM case where the right-hand side variable had a zero coefficient and $z_t = u_t$.

8.4 Further Developments

Work is currently being carried out on a number of developments of these models and the tools required to estimate them and test hypotheses about their parameters. The topic is less than a decade old, at the time of writing this chapter, but we hope to have demonstrated its potential importance.

A simple time-domain parameterization of the CM(ω, d) process has been developed in preliminary work by Abadir, Distaso, and Giraitis. The frequency-domain estimation of this process is also being considered, generalizing the FELW estimator of Abadir, Distaso, and Giraitis (2007) to the case where ω is not necessarily zero.

8.5 Acknowledgments

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References

Abadir, K. M., G. Caggiano, and G. Talmain. 2006. Nelson-Plosser revisited: the ACF approach. Working Paper Series 18-08 (revised version, 2008), Rimini Centre for Economic Analysis, Rimini, Italy.

Abadir, K. M., W. Distaso, and L. Giraitis. 2007. Nonstationarity-extended local Whittle estimation. *Journal of Econometrics* 141: 1353–1384.

Abadir, K. M., and G. Talmain. 2002. Aggregation, persistence and volatility in a macro model. *Review of Economic Studies* 69: 749–779.

Abadir, K. M., and G. Talmain. 2005. Autocovariance functions of series and of their transforms. *Journal of Econometrics* 124: 227–252.

Abadir, K. M., and G. Talmain. 2008. Macro and financial markets: the memory of an elephant? Working Paper Series 17-08, Rimini Centre for Economic Analysis.

Engle, R. F., D. F. Hendry, and J. -F. Richard. 1983. Exogeneity. *Econometrica* 51: 277–304.

Giraitis, L., J. Hidalgo, and P. M. Robinson. 2001. Gaussian estimation of parametric spectral density with unknown pole. *Annals of Statistics* 29: 987–1023.

Granger, C. W. J. 1980. Long memory relationships and the aggregation of dynamic models. *Journal of Econometrics* 14: 227–238.

Granger, C. W. J. 1981. Some properties of time series data and their use in econometric model specification. *Journal of Econometrics* 16: 121–130.

Granger, C. W. J., and R. Joyeux. 1980. An introduction to long-range time series models and fractional differencing. *Journal of Time Series Analysis* 1: 15–29.

Hidalgo, J. 2005. Semiparametric estimation for stationary processes whose spectra have an unknown pole. *Annals of Statistics* 33: 1843–1889.

Hosking, J. R. M. 1981. Fractional differencing. *Biometrika* 68: 165–176.

Nelson, C. R., and C. I. Plosser. 1982. Trends and random walks in macroeconomic time series: some evidence and implications. *Journal of Monetary Economics* 10: 139–162.

Robinson, P. M. 1978. Statistical inference for a random coefficient autoregressive model. *Scandinavian Journal of Statistics* 5: 163–168.

Tsay, W. -J., and C.-F. Chung. 2000. The spurious regression of fractionally integrated processes. *Journal of Econometrics* 96: 155–182.