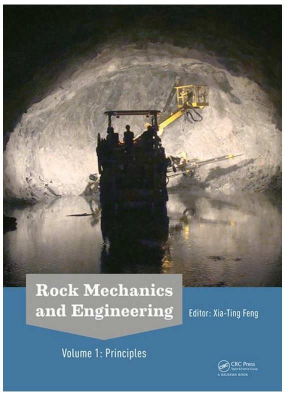

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Chapter 14

The MSDP$_u$ multiaxial criterion for the strength of rocks and rock masses

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Abstract: The natural and induced stresses near openings in rock media usually have different magnitudes along different directions. These have to be taken into account for stability analysis. A general multiaxial criterion was developed to assess the strength of rocks and rock masses. The MSDP$_u$ criterion was formulated in a modular manner, so it can be adapted to a variety of rock characteristics and structural calculations. The main equations are presented in this chapter. Specific conditions are described, including short term failure, initiation of damage, time effect, and the influence of scale and defects. In the case of rock mass, the criterion includes a continuity parameter that can be linked to the RMR geomechanical classification. The MSDP$_u$ criterion is compared with other formulations to highlight some of the similarities and differences. Validation and application of the criterion are illustrated using laboratory tests results; additional studies also present the analysis of borehole stability and failure around large scale excavations.

1 INTRODUCTION

The multiaxial stress state that exists in the rock around openings must be taken into account to assess their behavior and stability. Many criteria have been proposed to evaluate the possible occurrence of failure in rock media (*e.g.*, Jaeger & Cook, 1979; Franklin & Dusseault, 1989; Lade, 1993; Andreev, 1995; Sheorey, 1997) and other similar materials (Wastiels, 1979; Meredith, 1990; Theocaris, 1995; Yu, 2002; Papanikolaou & Kappos, 2007; Du *et al.*, 2010; Lu *et al.*, 2016). A variety of factors may affect rock failure, including scale, time, and characteristics of the defect population (from microcracks and pores to joints and faults).

The most popular expressions used in practice, such as the Coulomb (Lama, 1974; Goodman, 1980) and Hoek and Brown (1980a, 1980b, 1988, 1997) formulations, rely on equations that involve only the major σ_1 and minor σ_3 principal stresses, hence neglecting (or simplifying) the effect of the intermediate principal stress σ_2. Commonly used criteria also omit the influence of time and use simplified scaling parameters to go from laboratory specimen to rock mass size. When combined with idealized constitutive laws (such as linear elasticity), such simplifications may greatly help users faced with actual stability calculations, while reducing the effort needed to obtain the required material parameters. However, these simplifications can reduce significantly the accuracy of the calculations.

Detailed studies required for some of the most challenging rock engineering projects, such as large scale caverns, transportation tunnels, underground storage reservoirs, and toxic waste disposal facilities, typically require the use of more elaborate models developed over the years (*e.g.*, Desai & Salami, 1987; Aubertin *et al.*, 1994, 1998; Shao *et al.*, 1996; Cristescu & Hunsche, 1997). Such models must rely on representative expressions to define specific states, such as onset of cracking and ultimate strength. These are usually expressed in stress space using an appropriately formulated criterion.

Many relatively brittle materials (rocks, concrete, cast iron, ceramics) used by engineers share common features. For instance, their uniaxial compression strength C_0 largely exceeds their axial tensile strength T_0. Also, the maximum deviatoric stress that can be supported largely depends on the loading geometry, with all three principal stresses influencing their strength. The maximum load that can be supported is generally reached at small strain ($\varepsilon < 1\%$), and the stress-strain curve beyond the peak shows a significant drop (under a low confining pressure).

Failure of rock (and other brittle materials) is associated with the evolution of micro-cracks and appearance of macro-cracks associated with localized deformations. Failure thus constitutes a limit condition that engineers want to avoid. The failure condition is usually defined in stress space with a surface represented by a mathematical expression known as the failure criterion. This expression and the related parameters are generally obtained from laboratory tests performed under well-defined stress path. The maximum load supported by the tested specimen represents the failure strength. Performing several tests under different loading conditions gives different points on the failure surface. The selected mathematical expression is often (partially) empirical, based on a good adjustment of the formulation to experimental results. Many criteria have been developed for rocks and somewhat similar (brittle) materials (see reviews by Andreev, 1995; Sheorey, 1997; see also Aubertin *et al.*, 1999, 2000). Each of these criteria has its advantages and limitations, and none is universal or unanimously accepted.

The $MSDP_u$ criterion will be presented below, following a description of the main features that are deemed required for a rock failure criterion.

General description of rock failure

Like many other brittle materials, rocks often have a low porosity (usually less than a few percent). Their failure surface in the principal stress space, which represents the locus of the peak strength, is closed along the axes of negative (tensile) stresses and it remains open along the positive (compressive) stresses axes.

Before reaching failure, different stages are encountered (Paterson, 1978; Aubertin & Simon, 1997). For instance, a hard rock specimen submitted to an unconfined uniaxial compression test first shows a stage of tightening (or elastic contraction) due to the closure of micro cracks, followed by a stage of linearly elasticity, then a stage of inelasticity when the applied stress exceeds the threshold of crack propagation. This crack propagation can eventually drive the sample to failure at peak strength.

Such failure usually results from the propagation and eventual coalescence of micro-cracks (Li & Nordlund, 1993; Germanovitch *et al.*, 1996). The onset of crack propagation and their subsequent interaction leading to the failure depend on the path followed in the stress space. The application of compressive stresses, which tend to close some of the micro cracks, results in the mobilization of frictional resistance on the contact faces (McClintock & Walsh, 1962); this effect is related to the influence of the hydrostatic component of the stress tensor on the propagation threshold and on the ultimate strength of rocks.

Description of the failure envelope

The failure surface can be represented in the tridimensional space of principal stresses $\sigma_1, \sigma_2, \sigma_3$ (with $\sigma_1 \geq \sigma_2 \geq \sigma_3$). It can also be visualized in the octahedral (π) plane and in the $I_1 - J_2^{1/2}$ plane (where I_1 is the first invariant of the stress tensor σ_{ij}; J_2 is the second invariant of the deviatoric stress tensor S_{ij}).

Fig. 1 shows this surface, for the $MSDP_u$ criterion, in the conventional triaxial compression (CTC) test stress plane, $\sqrt{2}\sigma_x - \sigma_z$ (where $\sigma_x = \sigma_y$, represents the stress applied along the horizontal (radial) axes, and σ_z is the vertical stress). The shape of this criterion is based on physical and phenomenological considerations, which can be summarized by using specific points and segments identified on the curve of ultimate strength shown in Fig. 1.

When the specimen is submitted to uniaxial loading, its resistance is C_0 in compression and T_0 in tension ($T_0 < 0$); these two conditions are respectively represented by points A and B on Fig. 1. Point D represents a state of biaxial loading in tension, with $\sigma_x = \sigma_y < 0$ and $\sigma_z = 0$. In this case, the σ_x value should exceed T_0, *i.e.* $|\sigma_x| < |T_0|$ (*e.g.*, Theocaris, 1995; Aubertin & Simon, 1998). A state of spherical loading in tension ($\sigma_x = \sigma_y = \sigma_z < 0$) should also leads to a lower strength in absolute value than axial tensile loading, *i.e.* $|\sigma_x| < |T_0|$ (see point C). It is to be noted that several existing criteria, often inspired from the two dimensional criterion of Griffith (1921, 1924), consider that failure in tension can only be produced when one of the three principal stresses becomes equal to T_0. However, this vision is not supported by the physics of the problem or by a theoretical analysis of the failure conditions under multiaxial loading; when the stress component perpendicular to the critical failure plane is equal to T_0, the

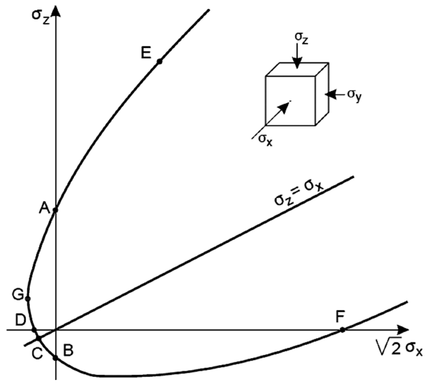

Figure 1 A schematic representation of the failure surface of rocks in the conventional triaxial plane (taken from Aubertin & Simon, 1996, 1998).

individual value of the principal stresses will be higher than T_0 (*i.e.* smaller than $|T_0|$ in absolute value) in the case of biaxial or spherical tensile loading. Therefore, such a Griffith type approach is not conservative, and it can lead to an overestimation of the strength of brittle materials submitted to these types of tensile loading. It also creates an apex (with a singularity) on the negative side of the stresses axes, which is problematic from a numerical point of view.

When one applies simultaneously some compressive and tensile stresses, as is the case between points D and A or between points B and F in Fig. 1, some criteria consider that the strength is only controlled by the highest tensile stress (in absolute value), with failure if $\sigma_z = T_0$ or $\sigma_x = T_0$, or by the maximum compressive stress that must be lower or equal to C_0. However, some laboratory test results, such as those reported by Andreev (1995, Figs. 6.175b and 6.176b) and Hunsche (1994, Fig. 3.19) indicate that the application of a relatively small compressive stress perpendicularly to the axis of tension can increase the material strength due to a tightening (increased stiffness) associated with closure of micro-cracks and mobilization of friction along the contact faces of cracks; this is happening around point G in Fig. 1. When the compressive stress becomes large enough, it also participates to (wing) crack propagation, leading to failure (Aubertin & Simon, 1998). Point F on Fig. 1 represents the strength of a material submitted to a biaxial compression, with $\sigma_x = \sigma_y > C_0$ (also shown by Ottosen, 1977; Maso & Lereau, 1980; Lade, 1982, 1993). Beyond point A (= C_0), all stresses are in compression, the propagation of closed cracks implies that the frictional resistance along contact faces must be overcome (McClintock & Walsh, 1962). The value of the available friction coefficient along the contact faces may change with the normal stress, due to the shearing of asperities as is the case with geological discontinuities (Patton, 1966; Ladanyi & Archambault, 1970). One thus expects that the slope of the failure criterion, which reflects the mobilization of friction, progressively decreases with the increased confining stress. This slope reduction ends when all asperities are sheared and the frictional sliding takes place over flattened surfaces. The contribution of this friction to the strength of the material becomes proportional to the residual friction angle ϕ_r (which is often close to the base friction angle ϕ_b). The slope of the failure criterion in the $\sqrt{2}\sigma_x - \sigma_z$ plane is then constant beyond point E in Fig. 1. The failure locus then corresponds to a linear (Coulomb type) criterion under higher mean stress, while for a lower means stress, the apparent friction angle tends to decrease progressively, as observed on rocks (Singh *et al.*, 1989; Charlez, 1991) and concrete (Chen & Chen, 1975).

The $MSDP_u$ failure criterion in the I_1–$J_2^{1/2}$ plane is defined by a curve on which the conventional triaxial compression (CTC) test strength (where $\sigma_z = \sigma_1 \geq \sigma_x = \sigma_y = \sigma_2 = \sigma_3$) is located above that of reduced triaxial extension (RTE) test (where $\sigma_z = \sigma_3 \leq \sigma_x = \sigma_y = \sigma_2 = \sigma_1$). This difference reflects the effect of the intermediate principal stress σ_2 on the ultimate strength of rock and similar materials, as demonstrated by various experimental observations (*e.g.*, Mills & Zimmerman, 1970; Akai & Mori, 1970). In this I_1–$J_2^{1/2}$ plane, the criterion is defined by a curve up to the point where it becomes a straight line of slope α in CTC (as shown in Fig. 2). In the π plane, the criterion forms a rounded triangle.

Many of the features described above are captured by the multiaxial criterion presented in the following sections.

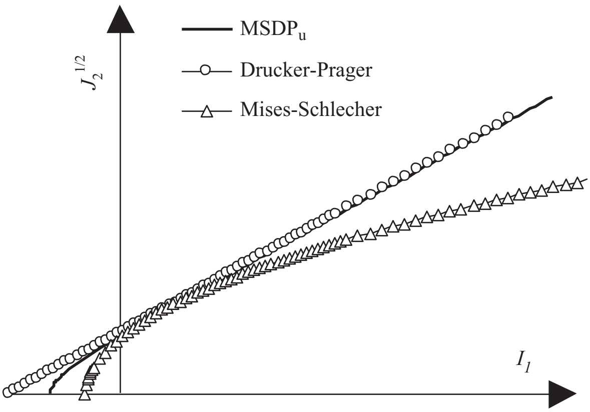

Figure 2 Schematical comparison between the $MSDP_u$ criterion and related criteria (taken from Li *et al.*, 2005).

2 THE MSDP$_U$ CRITERION FOR INTACT ROCK

The proposed 3D criterion, named $MSDP_u$ (which stands for Mises-Schleicher & Drucker-Prager unified), was developed to define a general unified locus in stress space. It has been formulated in a modular manner, so it can be adapted to a variety of rock and rock masses (and other media) for structural calculations. This criterion was initially developed to describe the short term strength of intact rock (Aubertin & Simon, 1996) and other low porosity, brittle materials (Aubertin & Simon, 1998). It has later been extended to describe the short term (Aubertin *et al.*, 1999) and long term strength of rocks (Aubertin & Simon, 1997) and rock masses (Aubertin *et al.*, 2000), considering various influence factors such as porosity and the presence of discontinuities at different scales. The main components of the criterion are presented below; other features are also mentioned in relation with additional applications described elsewhere.

The formulation of the $MSDP_u$ criterion for intact rocks is presented in Table 1 (left column). The basic equation for the failure strength can be written as:

$$\sqrt{J_2} = F_0 F_\pi \tag{1}$$

In this equation, F_0 is a function of the first stress invariant I_1. This function includes parameters α, a_1 and a_2 defined from basic material properties, *i.e.* σ_c and σ_t, the uniaxial compressive and tensile strength (in absolute values) respectively, and ϕ, the friction angle on plane surfaces ($\phi \cong \phi_r$, the residual friction angle). Parameter b reflects the ratio of the locus size at a Lode angle $\theta = -30°$ and $30°$ in the π plane (see Fig. 3); the value of b can range from about 0.7 to 1.

The $MSDP_u$ criterion adopts the shape of a rounded triangle in the octahedral (π) plane (Aubertin *et al.*, 1994), to represent the higher strength under triaxial compression than under reduced extension (Fig. 3b). An alternative expression has also been proposed for the formulation of F_π, so it can be reduced to the shape of an isosceles triangle in the π plane (Aubertin & Li, 2004; Li *et al.*, 2005); this formulation is better suited for granular materials and is not deemed required for rock media.

Table 1 Formulation of the $MSDP_u$ criterion.

For brittle intact rock	*For porous rock*†	*For damaged rock and rock mass*†
$\sqrt{J_2} - F_0 F_\pi = 0$	$\sqrt{J_2} - F_0 F_\pi = 0$	$\sqrt{J_2} - F_0 F_\pi = 0$
$F_0 = [\alpha^2(I_1^2 - 2a_1 I_1) + a_2^2]^{1/2}$	$F_0 = \left\{\alpha^2(I_1^2 - 2a_{1n}I_1) + a_{2n}^2 - a_{3n}\langle I_1 - I_{cn}\rangle^2\right\}^{1/2}$	$F_0 = \left[\alpha^2(I_1^2 - 2\widetilde{a}_1 I_1) + \widetilde{a}_2^2 - a_3'\langle I_1 - I_c\rangle^2\right]^{1/2}$
$F_\pi = b[b^2 + (1-b^2)\sin^2(45^\circ - 1.5\theta)]^{-1/2}$	$a_{1n} = \left(\frac{\sigma_{cn} - \sigma_{tn}}{2}\right) - \left(\frac{\sigma_{cn}^2 - (\sigma_{tn}/b)^2}{6\alpha^2(\sigma_{cn} + \sigma_{tn})}\right)$	$\widetilde{a}_1 = \Gamma a_1 = \left(\frac{\widetilde{\sigma}_c - \widetilde{\sigma}_t}{2}\right) - \left(\frac{\widetilde{\sigma}_c^2 - (\widetilde{\sigma}_t/b)^2}{6\alpha^2(\widetilde{\sigma}_c + \widetilde{\sigma}_t)}\right)$
$\theta = \frac{1}{3}\sin^{-1}\frac{3\sqrt{3}\ J_3}{2\sqrt{J_2^3}}; -30^\circ \le \theta \le 30^\circ$	$a_{2n} = \left\{\left(\frac{\sigma_{cn} + (\sigma_{tn}/b^2)}{3(\sigma_{cn} + \sigma_{tn})} - \alpha^2\right)\sigma_{cn}\sigma_{tn}\right\}^{1/2}$	$\widetilde{a}_2 = \Gamma a_2 = \left\{\left(\frac{\widetilde{\sigma}_c + (\widetilde{\sigma}_t/b^2)}{3(\widetilde{\sigma}_c + \widetilde{\sigma}_t)} - \alpha^2\right)\widetilde{\sigma}_c\widetilde{\sigma}_t\right\}^{1/2}$
$\alpha = \frac{2\sin\phi}{\sqrt{3}(3 - \sin\phi)}$	$a_{3n} = \frac{\alpha^2(I_{1n}^2 - 2a_{1n}I_{1n}) + a_{2n}^2}{(I_{1n} - I_{cn})^2}$	$a_3' = a_3(1 - \Gamma)$
$a_1 = \left(\frac{\sigma_c - \sigma_t}{2}\right) - \left(\frac{\sigma_c^2 - (\sigma_t/b)^2}{6\alpha^2(\sigma_c + \sigma_t)}\right)$		$\widetilde{\sigma}_c = \Gamma\sigma_c$ and $\widetilde{\sigma}_t = \Gamma\sigma_t$
$a_2 = \left\{\left(\frac{\sigma_c + (\sigma_t/b^2)}{3(\sigma_c + \sigma_t)} - \alpha^2\right)\sigma_c\sigma_t\right\}^{1/2}$		

†F_π, θ, and α have the same expressions as those for brittle intact rock

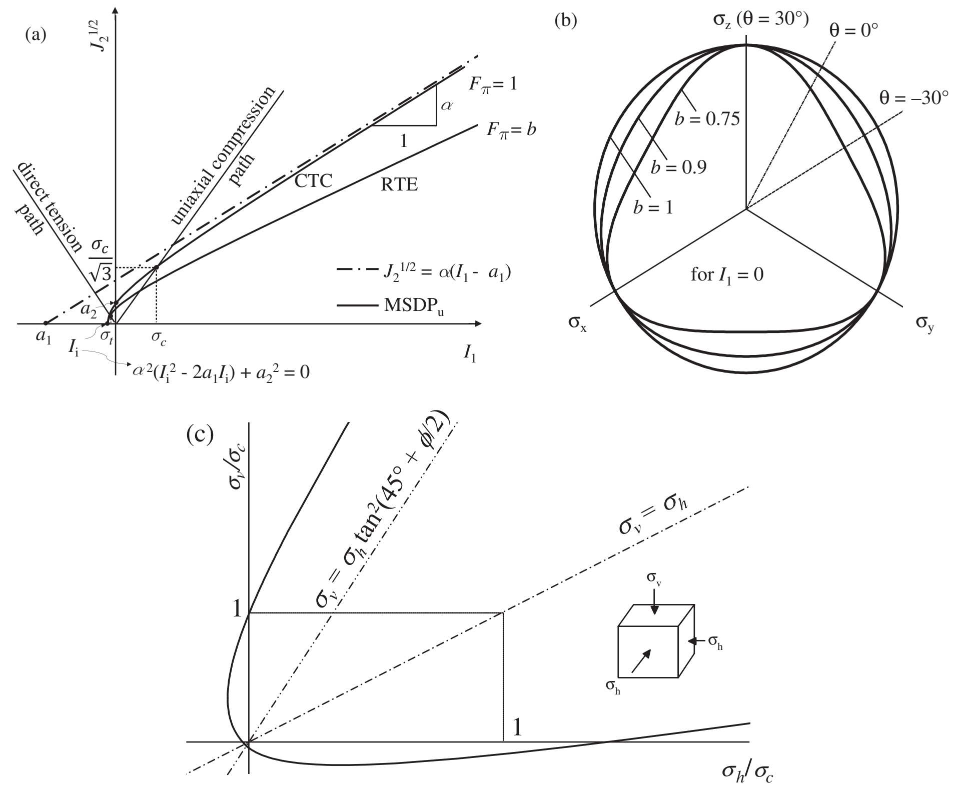

Figure 3 Schematic representation of the $MSDP_u$ criterion: a) in the $I_1 - J_2^{1/2}$ plane, CTC: conventional triaxial compression (θ = 30°), RTE: reduced triaxial extension (θ = −30°); b) in the octahedral plane; c) in the biaxial stress plane. Figures adapted from Aubertin *et al.* (2000).

Fig. 3c shows that, in the plane of the intermediate σ_2 and major σ_1 principal stresses (normalized by $C_0 = \sigma_c$), the shape of the criterion follows the conditions described above for Fig. 1.

The criterion can also be expressed in terms of the q and p invariants that are commonly used in soil mechanics (Li *et al.*, 2005).

In the $I_1 - J_2^{1/2}$ plane (Fig 3a), the failure envelope corresponding to the CTC condition is higher than that corresponding to the RTE condition. In the octahedral (π) plane (Fig. 3b), the shape of the $MSDP_u$ criterion can change from a circle (with b = 1) to a rounded triangle (for $1 > b \geq 0.70$). For most brittle rocks, the value of b is typically close to 0.75. In Fig. 3c, it can be seen that the biaxial compressive strength is higher than the uniaxial compressive strength (*i.e.* $\sigma_1 = \sigma_2 > \sigma_c$).

Fig. 3 indicates that for isotropic rocks submitted to conventional triaxial compression (CTC) tests, the $MSDP_u$ formulation practically reduces to the Mises-Schleicher criterion (Lubliner, 1990) at low mean stress and approaches the Drucker-Prager equation (Drucker & Prager, 1952; Desai & Siriwardane, 1984) at higher mean stress, as shown in Fig. 3a.

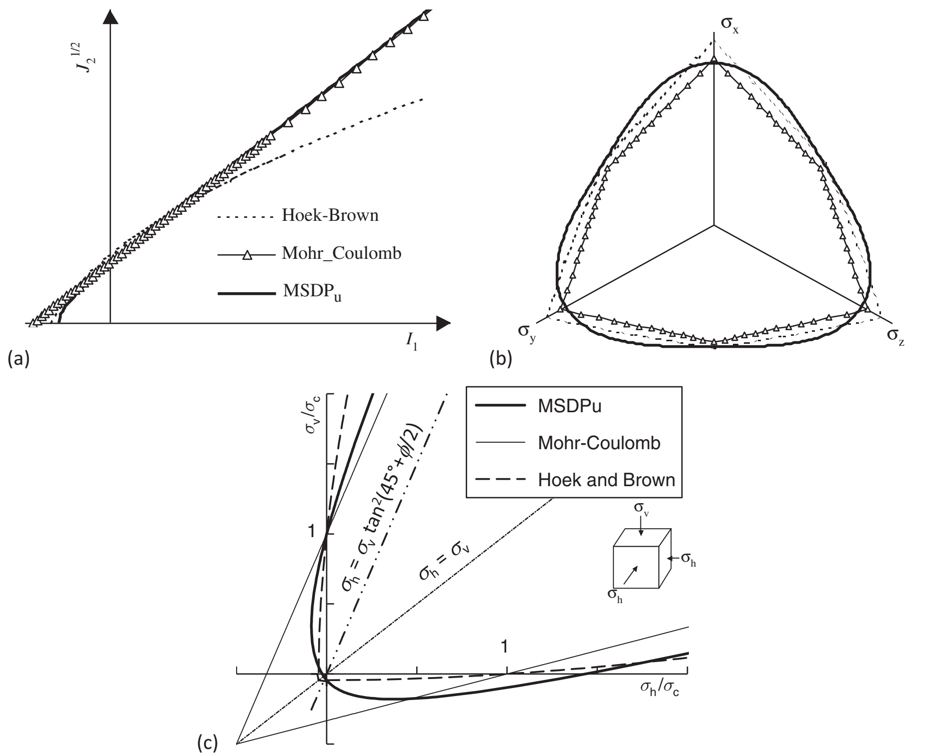

Figure 4 Schematic comparisons between the MSDP$_u$ criterion and two commonly used criteria for geomaterials (shown for normalized parameters): a) in the $I_1 - J_2^{1/2}$ plane; b) in the octahedral plane; c) in the biaxial stress plane.

Figure 4 shows a schematic representation of the MSDP$_u$ criterion (in the same three planes) plotted with the well-known Mohr-Coulomb and Hoek–Brown criteria. The MSDP$_u$ criterion is fairly close to these two criteria (Li *et al.*, 2005), but there are some important differences. For instance, the proposed criterion avoids the presence of a singularity on the tensile (negative stress) side, contrary to the Mohr-Coulomb and Hoek-Brown criteria; this apex may tend to overestimate the tensile strength of the materials because of the linear shape in the negative stresses quadrant (Fig. 4c). In the octahedral (π) plane (Fig. 4b) the MSDP$_u$ criterion takes a rounded triangle shape while the 3D Mohr-Coulomb and Hoek–Brown envelopes include singularities at $\theta = 30°$ and $-30°$. In the biaxial stress plane (Fig. 4c), the commonly used Mohr-Coulomb and Hoek–Brown formulations predict an equal strength under uniaxial compression and biaxial compression, contrary to the MSDP$_u$ criterion.

Also, the latter is based on the basic assumption that the uniaxial compression σ_c and tensile σ_t strengths are two distinct properties that are not directly related to each other, and which must be determined specifically (*e.g.* You 2015); the two other criteria shown in Fig. 4 consider that these two strength parameters are linked (*i.e.* one can thus be predicted using the other).

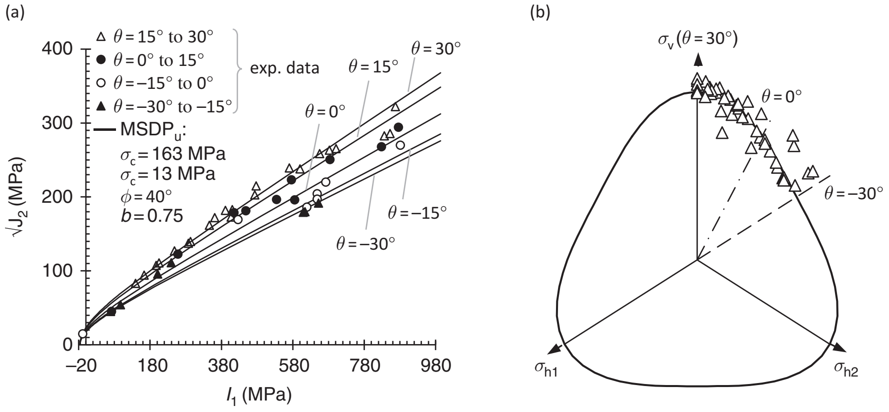

Figure 5 The MSDP$_u$ criterion applied to peak strength of Bowral trachyte: a) in $I_1 - J_2^{1/2}$ plane; b) in π plane. Tests results taken from Hoskins (1969); adapted from Aubertin *et al.* (1999).

The MSDP$_u$ criterion has been compared with various other criteria developed for geomaterials to illustrate the similarities and differences (Li *et al.*, 2005); these specific features will not be repeated here. It is nonetheless interesting to note that its shape is quite similar to the one described by Lundborg (1974) for the strength of intact rock submitted to multiaxial loading. The MSDP$_u$ characteristics are also quite close to those of other criteria recently developed for different materials (*e.g.*, Du *et al.*, 2010; Lu *et al.*, 2016).

The application of the MSDP$_u$ criterion to low porosity rock samples submitted to laboratory tests is straightforward and was illustrated in Aubertin *et al.* (1999, 2000), Li *et al.* (2000) and Li *et al.* (2005). Only the values of σ_c, σ_t, ϕ (= ϕ_r) and b are required to apply the criterion. These values are obtained from independent tests that can include uniaxial, diametric, and triaxial compression tests and shear tests on sheared surfaces. For homogeneous rocks, the full failure surface in stress space can be defined using a fairly small number of representative test results (depending on data scattering). It has previously been shown by the authors that the multiaxial formulation of MSDP$_u$ represents well different types of test results. For instance, Fig. 5 shows results obtained on Bowral trachyte.

3 TIME EFFECTS

All rocks may show a mechanical response that is time (or rate) dependent (Cristescu & Hunsche, 1997; Aubertin *et al.*, 1998). The proposed MSDP$_u$ criterion can be applied to define various stages of material failure.

Damage initiation and long term strength

Several studies have shown that a damage initiation threshold (DIT) exists in rock. It can be associated with the onset of micro-cracking, detected through volumetric strain measurements or acoustic emission activities (Paterson, 1978; Meredith, 1990; Martin & Chandler, 1994; Aubertin & Simon, 1997; Aubertin *et al.*, 1998). This threshold, which can also be seen as the long term strength of rocks, can be defined using the

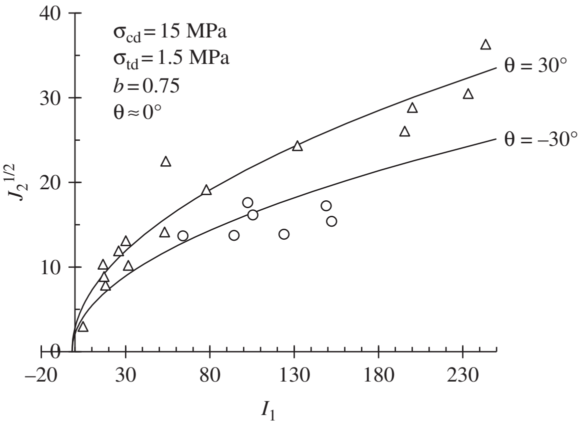

Figure 6 The $MSDP_u$ criterion applied to the damage initiation threshold (DIT) of rocksalt (data taken from Thorel, 1994). Figure adapted from Aubertin *et al.* (2000).

$MSDP_u$ criterion. For this type of application, parameters σ_c and σ_t are replaced by the corresponding values for the DIT (*i.e.* σ_{cd} and σ_{td}), which can be identified on the stress-strain curves. It has been observed that for many low porosity rocks, σ_{cd} = 0.3 to 0.7 σ_c and σ_{td} = 0.5 to 0.9 σ_t. As for parameters ϕ and b, their value does not seem to differ much when going from the usual short term to the long term (DIT) conditions.

The $MSDP_u$ criterion has been used to describe the long term failure surface of hard and brittle rocks as well as for softer and more ductile materials. For instance, Fig. 6 shows that the criterion matches fairly well (considering the data scattering) the results obtained on rocksalt samples tested by Thorel (1994), using a very low value of ϕ ($\phi_r \cong 0$) because of the viscous and plastic nature of rocksalt response at high mean stresses. For such semi-brittle behavior, $MSDP_u$ reduces to the Mises-Schleicher criterion for CTC tests, an expression frequently used for metals (Skrzypek & Hetnarski, 1993; Hjelm, 1994). When the mean stress is large enough, the surface becomes almost parallel to the I_1 axis, thus resembling the von Mises criterion (for b = 1).

Delayed failure

The strength of rock specimens submitted to a sustained deviatoric loading is expected to decrease over time. For a given stress state, the time to failure can be expressed with an simple equation based on an extension of Charles (1958) law for subcritical crack growth (Aubertin *et al.*, 2000). The equation can be formulated as:

$$t_f = \alpha_1 \left(\frac{\delta_1 + \delta_2}{\langle \delta_1 \rangle} \right)^{\beta} \tag{2}$$

where δ_1 is the difference between the applied deviatoric stress σ_{app} and the DIT and δ_2 is the difference between the short term strength STF (standard test) and σ_{app}; α_1, β are material parameters. Equation 2 can be used to evaluate strength as a function of time. For instance, Fig. 7a shows the application of this equation to Lac du Bonnet granite

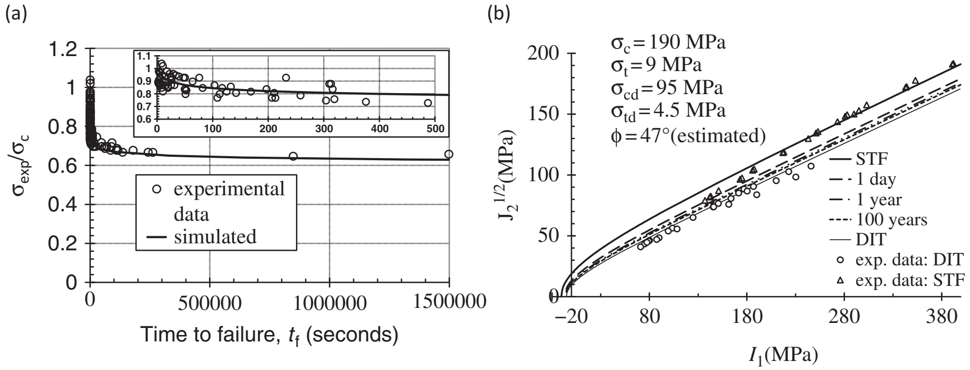

Figure 7 Time dependent failure strength of Lac du Bonnet granite cylinders: (a) uniaxial compression; STF tests performed on cylinders using a loading rate of about 1 MPa per second up to failure; other samples subjected to long term loading at constant loads (data taken from Schmidtke & Lajtai, 1985); (b) uniaxial and triaxial compression tests; application of MSDP$_u$ in $I_1 - J_2^{1/2}$ plane (data taken from Lau & Gorski, 1991). Figures adapted from Aubertin *et al.* (2000).

samples submitted to uniaxial compression tests, using data from Schmidtke & Lajtai (1985) with α_1 = 2.7 s, β = 9.73, and a long term uniaxial strength (DIT) taken as half the short term strength (STF). On this figure, it can be seen that the strength drop is initially more rapid and then progresses rather slowly toward the long term strength (DIT), which would be attained after a very long time. It is assumed here that any deviatoric stress below the DIT could be supported indefinitely (Aubertin & Simon, 1997; Aubertin *et al.*, 1998). Fig. 7b shows the corresponding isochronous curves for the strength of Lac du Bonnet granite for various time intervals; these curves are obtained by reducing σ_c and σ_t proportionally to the strength given by Equation 2 at a given time (data taken from Lau & Gorski, 1991).

4 SIZE EFFECT FOR INTACT ROCK

The strength of rock is influenced by scale, with the measured peak stress usually decreasing with sample size (Hoek & Brown, 1980b; Bieniawski, 1984; Cunha, 1993a, 1993b). This phenomenon has been linked mainly to statistical effects due to random strength and defect distribution (Jaeger & Cook, 1979) and to energy allocation and dissipation around cracks (Bazant & Planas, 1998). Size effect analysis is however complicated because it depends on the deformation processes, which in turn may vary with the loading state and testing method (Jaeger & Cook, 1979; Hudson & Harrison, 1997). Scale effects are usually more pronounced in very brittle materials, and they progressively decrease when going from a brittle to a semi-brittle behavior, altogether disappearing in the ductile (fully plastic) regime of inelastic flow. The influence of scale is also more pronounced in uniaxial tension than in uniaxial compression. It can be reduced substantially by applying a large confining pressure in triaxial compression tests.

Rock strength is decreased by larger defect size and defect density (Ramamurthy & Arora, 1994; Wong *et al.*, 1996). As the initial size of rock defects is often related to grain size, it can be expected that an increase in mean grain dimension also reduces failure strength (Wong *et al.*, 1996; Hatzor & Palchik, 1997).

Rock strength diminishes until the specimen size becomes equal to that of the large scale reference size d_L. Beyond this sufficiently large size, the effect of scale practically disappears. The strength then remains unchanged beyond d_L unless new types of defects are introduced (such as joint sets in a rock mass—see next section). The material strength σ_L on the scale of d_L can be much lower (< 25%) than the strength σ_S measured at the representative small scale d_S (typical of laboratory tested specimens), depending on the defect characteristics and loading state.

Various investigations have shown that the progressive decrease of strength can be related to the increasing size of the tested specimen using a power law function applied to the representative dimension (side, surface, volume; *e.g.* Jaeger & Cook, 1979; Cunha, 1993a, 1993b; Bazant & Chen, 1997; Hudson & Harrison, 1997). Such a scale effect function was proposed to define the strength from the smallest scale of the representative volume element d_S with a maximum strength σ_S, to the scale d_L where increasing the size does not affect strength any more (*i.e.* σ_L is the nominal large scale strength); this expression can be written as (Aubertin *et al.*, 2000, 2002):

$$\sigma_N = \sigma_S - x_1(\sigma_S - \sigma_L) \left\langle \frac{d_N - d_S}{d_L - d_S} \right\rangle^{m_1} \tag{3}$$

The first term on the right hand side is the strength σ_S at small scale d_S, and the second term represents the decreasing value as size increases until σ_L is reached at d_L. The rate at which the decrease takes place depends on two material parameters x_1 and m_1. In this equation, d_L is the reference size (length L, area L^2 or volume L^3) which has the minimum asymptotic strength σ_L, and d_S is the corresponding size when strength is considered maximum for a homogeneous representative volume element of the material. For rocks, the authors have proposed using $d_S \cong 10^y\, d_g$ and $d_L \cong 10^{2y}\, d_S$, where d_g is the mean grain size; here $y \cong 1$ for measures of length, $y \cong 2$ for area, and $y \cong 3$ for volume. In many practical cases, one finds that $d_S \cong 0.5^y$ to 5^y (cm, cm^2, cm^3), and typically $d_L \geq 10^2$ cm, 10^4 cm^2, 10^6 cm^3. $\langle\ \rangle$ are Macaulay brackets ($\langle x \rangle = (x + |x|)/2$), which limits the decrease of strength for $d \geq d_L$.

It can be noted here that an alternate equation (not presented here) has also been proposed by Aubertin *et al.* (2002) to represent a more progressive (and somewhat more representative) reduction of the strength with size.

A practical application procedure has been developed for Equation 3, based on statistical analysis of standard laboratory tests results. This has led to the following simple predictive equation to estimate the large scale strength of intact rocks (Aubertin *et al.*, 2001; Li *et al.*, 2001):

$$\sigma_L = z_1(\overline{\sigma}_{c50} + z_2 \times S_0) \tag{4}$$

where $\overline{\sigma}_{c50}$ is the average observed mean value of the uniaxial compressive strength on standard size specimens (50 mm), S_0 is the corresponding standard deviation of the test results (when at least 10 tests results are available), and z_1 and z_2 are two statistically obtained parameters (see details in Aubertin *et al.,* 2001). This equation was applied by Li *et al.* (2001) for the analyses of the URL tunnel (in Manitoba, Canada), with $z_1 \cong$ 0.08 and $z_2 \cong 5$ to 6.

Li *et al.* (2007) later proposed a statistical approach to estimate the value of σ_S from standard laboratory tests results on relatively hard rocks. The results from this

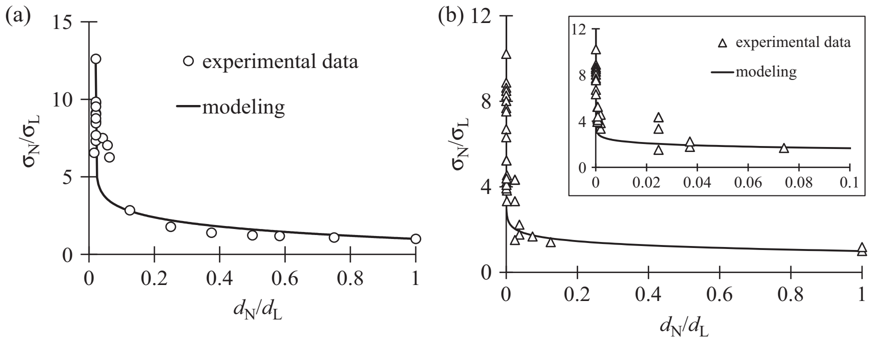

Figure 8 Influence of size on the uniaxial compressive strength: a) cubic coal specimens; x_1 = 1, m_1 = 0.075, d_L = 121.92 cm, σ_L = 4.48 MPa, d_S = 2.54 cm and σ_S = 56.54 MPa (data taken from Bieniawski, 1968); b) Cedar City quartz diorite with prismatic and cylindrical specimens; x_1 = 1, m_1 = 0.025, d_L = 2.67×10^5 cm^3, σ_L = 6.83 MPa, d_S = 28.4 cm^3 and σ_S = 86.7 MPa (data taken from Pratt *et al.*, 1972). Taken from Aubertin *et al.* (2000).

investigation indicate that the standard unconfined compressive strength σ_c is often close to about two thirds of the small scale unconfined compressive strength σ_S. This analysis suggests that the large scale strength of low porosity rocks can be as low as 20 % (or even less) of this standard strength.

Fig. 8 shows Equation 3 applied to test results presented by Bieniawski (1968) on coal (Fig. 8a) and by Pratt *et al.* (1972) on quartz diorite (Fig. 8b).

It is important to recall here that size effects are related to the presence and influence of defects (at various scales) on the behavior of rock. Increasing the confining pressure tends to reduce the influence of existing flaws, so it can also reduce size effects. This phenomenon has been illustrated, for instance, by the experimental results from Gerogiannopoulos & Brown (1978) on intact and granulated marble, and by measurements made for elastic properties and failure strength by Michelis (1987) and Medhurst & Brown (1998). This is also in accordance with the strength envelope of joints and intact rock which tends to converge at high normal stresses (Ladanyi & Archambault, 1970; Gerard, 1986). This factor however is not easily taken into account, and it has been largely neglected in previous scale effect investigations.

An approach was proposed to address this aspect with MSDP$_u$. To do so, parameters σ_t and σ_c are taken as variables whose values are corrected for scale and for stress state. The ensuing values of σ_{ts} and σ_{cs} are expressed according to Equation 3, with x_1 given by the following phenomenological function:

$$x_1 = \exp(x_0\,\sigma_3/T_0) \tag{5}$$

where T_0 is the uniaxial tensile strength σ_t (with a negative value) of standardized size specimens; T_0 is used here as normalizing parameter because it corresponds to the stress state where scale effects are near their maximum. Fig. 9a shows a schematic representation of the MSDP$_u$ criterion with Equation 3 used for σ_{ts} and σ_{cs}, with x_1 taken as a constant (x_1 = 1 or x_0 = 0, *i.e.* no effect of the stress state) or given by Equation 5 with x_0 > 1.

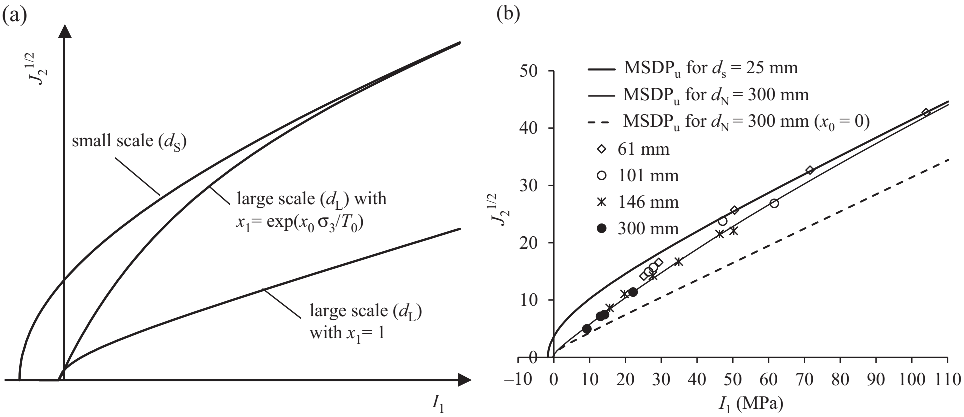

Figure 9 MSDP$_u$ criterion for different specimen sizes of intact rock: a) schematic representation of the MSDP$_u$ criterion for different specimen sizes of rock (x_0 = 0.5); b) application of MSDP$_u$ criterion to triaxial compression tests on large size cylindrical samples; description with x_0 = 0.065, m_l = 0.015, d_L = 120 cm, d_S = 2.5 cm, σ_{cS} = 35.0 MPa, σ_{cL} = 3.5 MPa, σ_{tS} = 1.5 MPa, σ_{tL} = 0.15 MPa (s_{cS} and s_{cL} are the uniaxial compression strengths obtained on small and large size specimens, respectively; s_{tS} and s_{tS} are the uniaxial tensile strengths of small and large specimens, respectively; data taken from Medhurst & Brown, 1998). Figures taken from Aubertin *et al.* (2000).

The effect of size theoretically disappears only when a fully plastic behavior is encountered (often at very high mean stress). For practical calculations with low porosity rocks, it can be considered, as a first estimate, that this effect becomes negligible when the shear strength of a closed defect (microcrack) surface becomes equal to the cohesion of the surrounding rock material. Based on the Coulomb criterion, this condition can be approximated by the following expression (Aubertin *et al.*, 2000):

$$\sigma_3 = \frac{\sigma_c}{2\tan\phi\,\tan(45^\circ + \phi/2)} \tag{6}$$

which gives $\sigma_3 \cong 0.5\ \sigma_c$ for $\phi \cong 30°$. Above this value of the confining pressure, scale effect becomes much less important, and the failure envelope at small scale d_S and large scale d_L tend to converge (see Fig. 9a). Fig. 9b shows how this concept applies to actual test results on rock samples of different sizes (data from Medhurst & Brown, 1998). Fig. 10 illustrates the effect of scale and of loading state on material strength according to Equations 3 and 5.

The complex influence of scale and stress state may explain why rock strength close to the walls of large underground openings may appear to be much lower than the value deduced for locations deeper in the rock mass, where the confining stresses are more significant.

5 YIELDING AND FAILURE OF POROUS ROCKS

It has long been known that rocks with a relatively high porosity typically show some inelastic yielding under a high mean stress, even with little or no deviatoric stress (Nova, 1986; Brown & Yu, 1988; Charlez, 1991; Shao & Henry, 1991; Abdulraheem

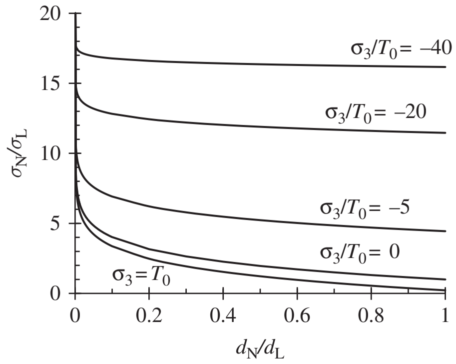

Figure 10 Representation of size effect with different confining pressures according to Equations 3 and 5 with $T_0 = -\sigma_t$, using $x_0 = 0.04$, $m_1 = 0.075$, $d_L = 1000$, $\sigma_L = 10$, $d_S = 1$, $\sigma_S = 200$ (taken from Aubertin *et al.* 2000).

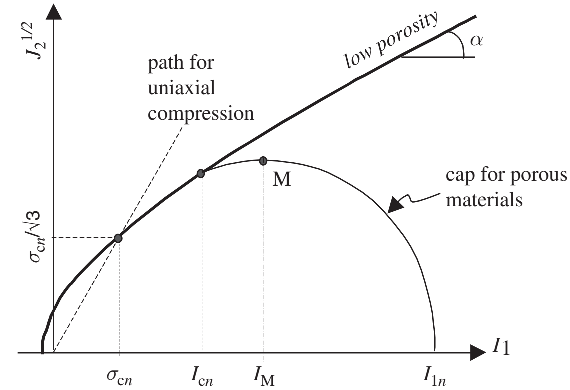

Figure 11 Schematic representation of the MSDP$_u$ criterion for dense and porous materials under CTC conditions ($\theta = 30°$); with key parameters. The maximum value of $J_2^{1/2}$ corresponds to point M (taken from Li *et al.*, 2005).

et al., 1992). The corresponding yield surface (which is more or less equivalent to a DIT) can then curve downward and eventually close on itself on the compressive side of the stresses. A "cap" can be used to capture the curvature under a large hydrostatic stress component (as is commonly done in soil mechanics; *e.g.* Desai & Siriwardane, 1984).

This approach has been applied to the MSDP$_u$ criterion by adding the last term on the right hand side of the formulation given in the central column in Table 1. The corresponding shape with (and without) this cap is shown in Fig. 11.

Modifications were later introduced in the MSDP$_u$ criterion to describe the yield or failure conditions in terms of porosity (Aubertin & Li, 2004; Li *et al.*, 2005). In this

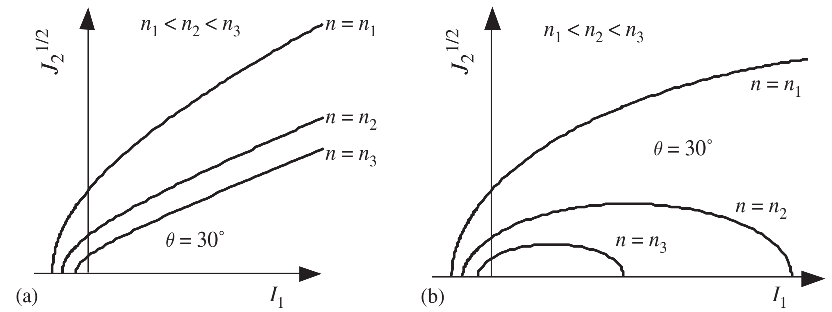

Figure 12 Schematic representation of the $MSDP_u$ criterion (a) for low porosity materials with $a_{3n} = 0$ and (b) for porous materials with $I_c \cong 0$, $a_{3n} \neq 0$, $b = 0.75$ (taken from Li *et al.*, 2005).

case, material parameters are expressed explicitly as a function of initial porosity using the following (Li & Aubertin, 2003):

$$\sigma_{un} = \left\{ \sigma_{u0} \left(1 - \sin^{x'_1} \left(\frac{\pi}{2} \frac{n}{n_C} \right) \right) + \langle \sigma_{u0} \rangle \cos^{x'_2} \left(\frac{\pi}{2} \frac{n}{n_C} \right) \right\} \left\{ 1 - \frac{\langle \sigma_{u0} \rangle}{2\sigma_{u0}} \right\} \tag{7}$$

where σ_{un} may be used for compression ($\sigma_{un} = \sigma_{cn}$) or tension ($\sigma_{un} = \sigma_{tn}$); n_C is the critical porosity for which σ_{un} tends toward zero, in tension ($n_C = n_{Ct}$) and in compression ($n_C = n_{Cc}$); parameter σ_{u0} represents the theoretical value of σ_{un} for $n = 0$; x'_1 and x'_2 control the non-linearity of the σ_{un}–n relationship.

With the cap, the criterion closes down toward the I_1 axis; parameter I_{cn} represents the I_1 value where the locus departs from the "low porosity" condition (see Fig. 11), while I_{1n} corresponds to the intersection of the criterion with the positive I_1 axis (also shown on Fig. 11).

The values of I_{cn} and I_{1n}, which may be obtained experimentally, become very large for dense materials; the Cap portion can then be neglected.

Specific functions have been developed to define I_{1n} and I_{cn} as a function of porosity n (Li *et al.*, 2005).

The effect of porosity on the criterion is illustrated in Fig. 12a for a typical high strength rock at low mean stress conditions (*i.e.* $I_1 < I_{cn}$), while Fig. 12b shows its shape for a relatively low strength material.

Fig. 13 show the locus with $b = 0.75$ defined for I_1 that extends beyond I_{cn}, in the case of rocks. The presence of the Cap is required in such cases to describe the elastic limit and the failure strength.

These results highlight the great flexibility of the proposed set of equations, which allow a good description of the inelastic loci and failure strength of rocks and various other geomaterials. Additional illustrations are presented in Aubertin *et al.* (2003), Aubertin & Li (2004), and Li *et al.* (2005).

It is also possible to predict the failure strength of rock for different porosities, when data is available for a given value of n, as demonstrated by Li *et al.* (2005) for sandstone.

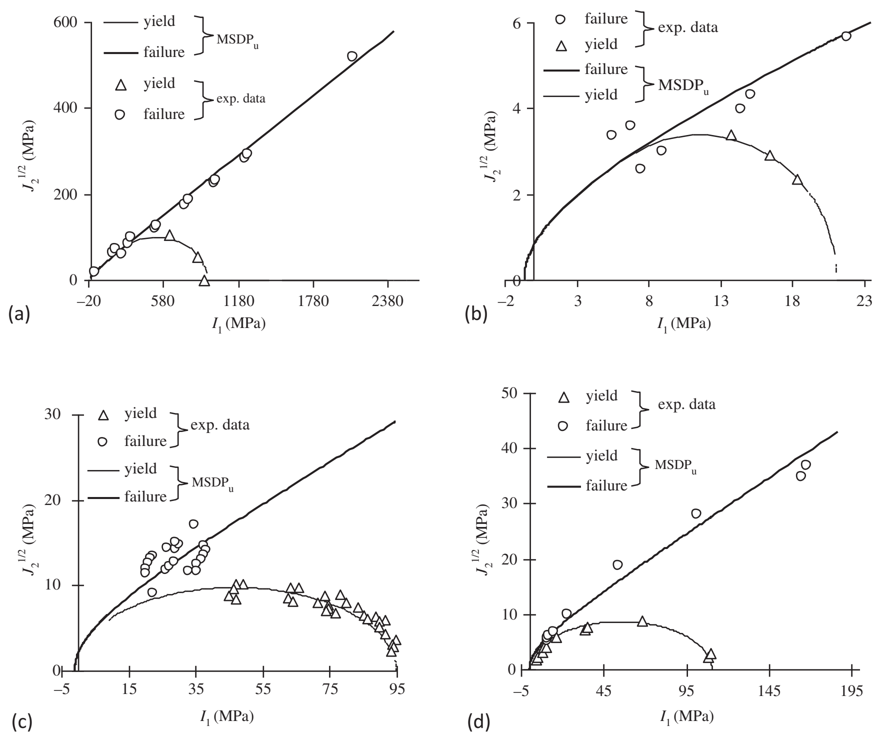

Figure 13 Failure strength and elastic limit (in CTC) of (a) Kayenta sandstone (data from Wong *et al.*, 1992), (b) a tuff (data from Pellegrino, 1970; figure adapted from Aubertin *et al.* 2000), (c) Bath stone (data from Elliott & Brown, 1985), and (d) Castlegate sandstone (data from Coop & Willson, 2003). For Kayenta sandstone, the MSDP$_u$ criterion was applied with ϕ = 30° (estimated), σ_{cn} = 30 MPa (measured), σ_{tn} = 2 MPa (estimated), and a_{3n} = 0 (or I_{cn} >>) for failure and with ϕ = 30° (estimated), σ_{cn} = 30 MPa (measured), σ_{tn} = 2 MPa (estimated), a_{3n} = 0.115 (estimated), and I_{cn} = 250 MPa (estimated) for yield. For the tuff, the MSDP$_u$ criterion was applied with ϕ = 20° (estimated), σ_{cn} = 3.8 MPa (measured), σ_{tn} = 0.5 MPa (estimated), and a_{3n} = 0 (or I_{cn} >>) for failure and with ϕ = 20° (estimated), σ_{cn} = 3.8 MPa (measured), σ_{tn} = 0.5 MPa (estimated), a_{3n} = 0.115 (estimated), and I_{cn} = 6.5 MPa (estimated) for yield. For Bath stone, the MSDP$_u$ criterion was applied with ϕ = 30° (estimated), σ_{cn} = 15 MPa (measured), σ_{tn} = 1 MPa (estimated), and a_{3n} = 0 (or I_{cn} >>) for failure and with ϕ = 30° (estimated), σ_{cn} = 15 MPa (measured), σ_{tn} = 1 MPa (estimated), a_{3n} = 0.095 (estimated), and I_{cn} = 0 MPa (estimated) for yield. For Castlegate sandstone, the MSDP$_u$ criterion was applied with ϕ = 26° (estimated), σ_{cn} = 9 MPa (estimated), σ_{tn} = 0.1 MPa (estimated), and a_{3n} = 0 (or I_{cn} >>) for failure and with ϕ = 26° (estimated), σ_{cn} = 9 MPa, σ_{tn} = 0.1 MPa (estimated), a_{3n} = 0.064 (estimated), and I_{cn} = 1 MPa (estimated) for yield.

6 APPLICATION TO DAMAGED ROCK AND FRACTURED ROCK MASS

The strength of rocks and rock masses depends on the initial structural state that can be represented by a continuity parameter Γ (described below). The introduction of parameter Γ into the formulation of the MSDP$_u$ criterion is shown in the right hand side

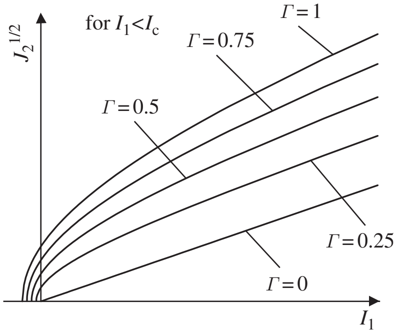

Figure 14 Schematic representation of the influence of the continuity parameter Γ on the failure surface of a tight undisturbed rock media (with $I_1 < I_c$); $\Gamma = 1$ corresponds to undamaged material, and $\Gamma = 0$ refers to a cohesionless media (taken from Aubertin *et al.* 2000).

column of Table 1 (Aubertin *et al.*, 2000). This continuity parameter can be seen as an equivalent damage parameter D (= 1 – Γ), as defined in the Kachanov-Rabotnov approach forming the basis for Continuum Damage Mechanics (Lemaitre, 1992). It is treated here as a scalar, although it could be extended to deal with anisotropy under a tensorial format (Aubertin *et al.*, 1998).

When the rock has few defects (*i.e.*, a very small population of cracks and pores), $\Gamma \cong 1$. When defects (flaws of various sizes) become more abundant and their influence more important, the value of Γ is reduced. For a highly fractured but relatively dense medium that behaves as a cohesionless soil, the value of Γ becomes nil; this means that $\tilde{a}_1 = \tilde{a}_2 = 0$ and $a'_3 = a_3$ in the F_0 equation (Table 1, right column). The proposed criterion then becomes equivalent to the Coulomb criterion without cohesion (for $I_1 \leq I_c$), as shown in Fig. 14. This figure also illustrates how the value of Γ influences the position of the surface in the $I_1 - J_2^{1/2}$ plane; it shows that the strength is reduced as Γ decreases. This effect of the continuity parameter can be combined with that of porosity (Aubertin *et al.*, 2000), as is shown in the following illustration.

Applications of $MSDP_u$ to results obtained on porous rocks and rock-like materials are shown in Fig. 15 (using data from Nguyen, 1972 and Wong *et al.*, 1992). As can be seen, the $MSDP_u$ is able to properly represent the behavior of these porous rocks, when considering also the effect of the continuity parameter.

This approach can also be applied to rock masses, although going from the behavior of rock to that of the large scale rock mass is quite a challenge.

The continuous scale effect described above for rocks applies when there is no new type of flaws introduced in the media. However, going from intact rock to in situ rock mass implies not only the usual statistical and energy release size effects, but also the addition of other types of larger scale defects such as joint sets. Thus, the relationship used for scaling up properties may become more or less discontinuous. This phenomenon is schematically illustrated in Fig. 16, which shows that the scale effect function can be considered continuous until new types of defects are introduced; this is the case with grain boundaries (grains to rock) and joint sets (rock to rock mass). In the transition zone (shade areas), the strength–size function becomes ill defined

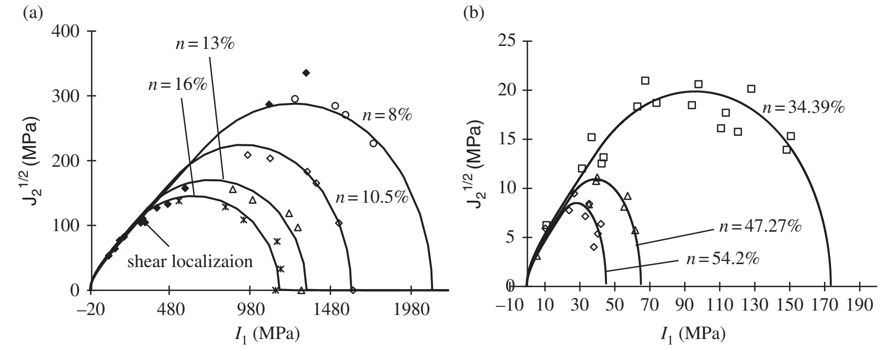

Figure 15 Applications of $MSDP_u$ criterion to porous materials: a) saturated Berea sandstone cylinders submitted to CTC loading condition (data taken from Wong *et al.*, 1992); description of test results using: $b = 0.75$, $\phi = 35°$, $\sigma_c = 110$ MPa, $\sigma_t = 9$ MPa, $a_3 = 0.75$; for $n = 8\%$, $\Gamma = 0.65$ and $I_c = 710$ MPa; for $n = 10.5\%$, $\Gamma = 0.75$ and $I_c = 515$ MPa; for $n = 13\%$, $\Gamma = 0.79$ and $I_c = 325$ MPa; for $n = 16\%$, $\Gamma = 0.8$ and $I_c = 250$ MPa; b) application to Paris plaster cylinders (with water to plaster ratio of 70%), representation with $b = 0.75$, $\phi = 33°$, $\sigma_c = 10$ MPa, $\sigma_t = 0.5$ MPa, $a_3 = 0.75$ (data taken from Nguyen, 1972); $\Gamma = 0.825$ and $I_c = 40$ MPa for $n = 34.39\%$; $\Gamma = 0.675$ and $I_c = 25$ MPa for $n = 47.27\%$; $\Gamma = 0.575$ and $I_c = 20$ MPa for $n = 54.2\%$. Figure adapted from Aubertin *et al.* (2000).

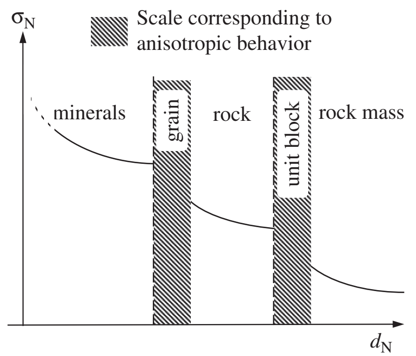

Figure 16 Schematic representation of scale effect on rock properties; the shaded areas represent scales at which strength is not isotropic. Figure taken from Aubertin *et al.* (2000).

because the material cannot be considered homogeneous and anisotropy needs to be considered.

Anisotropy is also a key aspect when only one or two defect families or few individual members are added. In this case, one must either use an anisotropic criterion or combine an isotropic expression (such as $MSDP_u$) with a shear strength criterion for the weakness planes (Li & Aubertin, 2000).

Rock masses with more than two distinct joint sets usually behave almost isotropically on a large scale. It is thus often considered that an isotropic criterion is appropriate for such cases (Hoek & Brown, 1980a, 1980b, 1988).

One of the challenges associated with defining the strength of a rock mass stems from the difficulty to perform appropriate tests and obtain adequate in situ rock mass properties (strength and deformability) at a scale corresponding to engineering structures. Consequently, the commonly used approach in rock mechanics has been to rely on laboratory properties "corrected" for scale and discontinuity conditions. For that purpose, several techniques have been proposed, with each technique suffering from some limitations and fairly large uncertainties.

The approach applied here is based on the use of the continuity parameter, introduced above. The value of Γ can be related to the reduction of strength parameters when compared to "undamaged" materials.

For strength and stability calculations, the continuity parameter Γ can be used as a correction factor acting on "undamaged" material properties (Aubertin *et al.*, 2000).

Parameter Γ of an isotropic rock mass can be defined from either the deformability or the strength. Its effect is to alter the stress state in the bearing portion areas, which in turn decreases proportionally all mechanical properties. For this application of the $\mathrm{MSDP_u}$ criterion to rock mass strength, the authors have proposed the following expression:

$$\Gamma = \Gamma_{100}\left[0.5\left(1 - \cos\frac{\pi \mathrm{RMR}}{100}\right)\right]^{p'} \tag{8}$$

with

$$\Gamma_{100} = \sigma_{\mathrm{cL}}/\sigma_{\mathrm{c}} \tag{9}$$

Here, σ_{cL} is taken as the uniaxial compressive strength of the rock at size d_{L} (see Equations 3 and 4), while σ_{c} is the standard size specimen strength of intact rock. This equation was based on an expression proposed by Mitri *et al.* (1994) for the deformability of rock masses; exponent p' was added to better represent strength parameters. In Equation 9, the value of σ_{cL} is usually found to be 0.2 to 0.3 times σ_{c}; σ_{cL} can be seen as the rock mass strength when RMR is 100 (n.b. the Bieniawski, 1989 RMR version is used).

Fig. 17 shows this relationship with Γ_{100} = 0.3 and p' = 1, 2 and 3 (a value of 3 is favored for practical calculations). Also shown in this figure is the $s^{1/2}$ parameter (= $\sigma_{\mathrm{cmass}}/\sigma_{\mathrm{c}}$) expressed from the relationship proposed by Hoek and Brown (1997). The two functions are fairly close to each other at RMR values below about 80, but differ at larger RMR. Here, Γ at high RMR values is bounded by Γ_{100} corresponding to the large scale strength of the rock σ_{cL}.

The value of parameter Γ given by Equation 8 can be introduced into the general $\mathrm{MSDP_u}$ criterion, with values of σ_{c} and σ_{t} given for standard size samples. Alternately, one could use Γ_{100} = 1 in Equation 8 and use σ_{c} and σ_{t} values in the F_0 equation (Table 1, right column) corresponding directly to large scale conditions (from Equations 3 and 5).

Figure 18 shows the failure strength envelope (for $I_1 < I_{\mathrm{c}}$) using Γ obtained from Equation 8 for different RMR values; note that the influence of loading mode (Equation 4) is neglected in this representation. As expected, reducing the RMR decreases the rock mass strength; a highly fractured mass may even behaves as a purely frictional (cohesionless) material.

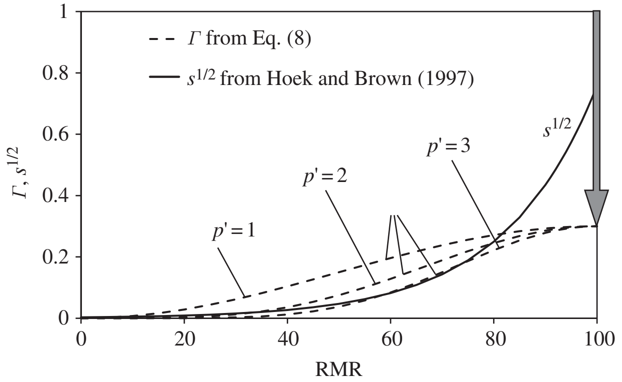

Figure 17 Continuity parameter Γ vs. rock mass rating (RMR)1989; also shown is the $s^{1/2}$ value as described by the relationship proposed by Hoek and Brown (1997). The vertical arrow on the right hand side shows the effect of scale on intact rock strength (going from the specimen size to the large scale unit volume). Figure taken from Aubertin *et al.* (2000).

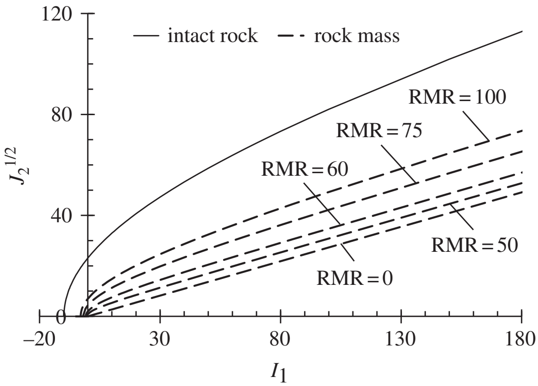

Figure 18 The MSDP$_u$ criterion for rock and rock masses (for $I_1 < I_c$) according to Equation 8 with b = 0.75, ϕ = 35°, σ_c = 220 MPa, σ_t = 9 MPa, Γ_{100} = 0.3, p' = 3 for various RMR values (taken from Aubertin *et al.* 2000).

7 DISCUSSION

The application of the MSDP$_u$ criterion for the strength analysis of rock and other somewhat similar materials has been documented in many of the publications mentioned above. These include specific conditions not explicitly addressed here, such as the response of cohesionless or weakly cemented porous materials, and rocks with planar anisotropy.

The criterion has also been used to conduct analysis of engineered openings in rock media, as illustrated in several publications. These cannot be presented here due to space limitation; some key examples are nonetheless recalled in the followings.

The stability analysis of boreholes in isotropic rock can be performed directly using laboratory properties. This was shown using experimental results of Mazanti &

Sowers (1965) obtained from hollow cylinder tests on granite (Aubertin *et al.*, 2000). The $MSDP_u$ criterion has also been used to back-calculate in situ stresses around boreholes using breakout geometry (Li & Aubertin, 1999; Li *et al.*, 1999, 2000; Kikiessa-Kisaka, 2015).

Simulations of the short term failure and delayed failure around a large scale tunnel at the AECL underground facilities (Manitoba, Canada) has been described by Li *et al.* (2000) and Aubertin *et al.* (2000).

The $MSDP_u$ criterion equations have also been used to develop analytical solutions for the stresses around circular openings in an elasto-plastic media, with and without the Cap component (Li *et al.*, 2005, 2010; Li & Aubertin, 2009). These original solutions have been successfully compared with other specific formulations (encompass by $MSDP_u$) for the Coulomb and von Mises criteria; validation was also conducted using plane strain numerical simulations.

The implementation of the $MSDP_u$ criterion in FLAC (Itasca) as part of a new elasto-plastic constitutive model has been presented by Li *et al.* (2010); the latter includes the application of the ensuing $MSDP_u$–EP model to determine the stresses in backfilled mine stopes.

These applications have shown that the multiaxial criterion presented in the chapter is simple to use, and can be applied for a wide variety of rock media characteristics and loading conditions. Like any other criterion, it also has some limitations, some of which are briefly recalled in the following.

i) The criterion equations presented above are only applicable to isotropic media, so it cannot be applied to the shaded areas shown schematically in Fig. 16. This is also the case for most existing criteria. As mentioned however, it has also been adapted for planar anisotropy (Li & Aubertin, 2000), but this aspect needs to be investigated further.
ii) Size effects involve complex physical phenomena, and the strength magnitude at various scales depends on the controlling inelastic processes leading to failure (or yielding). With low porosity rocks, scale effects tends to be reduced when the mean stress is increased because the added confinement diminishes the influence of existing flaws by closing the opened crack faces (see Equation 4). However, not all flaws (microcracks to joints) will be perfectly matched upon closure, so it can be expected that scale effects cannot be fully eliminated simply by increasing the mean stress, especially when natural porosity increases. More work needs to be done on this aspect, especially to assess scale effects and other influence factors in the damaged rock zone around openings where the stress distribution is highly non-uniform.
iii) It is assumed here that rock strength (from DIT to STF) is related to the initial defect state. However, additional provisions are required to include the evolution of the damage state. An internal state variable approach can be used to treat complex load paths and history, including the effect of progressive damage growth (Aubertin *et al.*, 1994, 1998). For most practical calculations however, the simplified procedure presented in the referenced papers, commonly in rock engineering, provides a good estimate of failure occurrence.

This presentation has not taken into account the intrinsic variability of rock properties. As with any other criteria, this aspect should be treated adequately with $MSDP_u$, by using proper statistical tools.

Despite these limitations, the sound physical basis from which it has been formulated, its unified and modular nature, and its adaptability to treat hard and soft materials with little ($\Gamma \cong 1$) or many flaws ($0 < \Gamma < 1$) make the proposed criterion a practical engineering tool.

8 FINAL REMARKS

In this chapter, the authors have presented a summary of the main features for the general multiaxial criterion $MSDP_u$, initially developed for intact rock samples and extended for different types of rock media. The proposed criterion can be applied to describe the short term strength and the damage initiation threshold (DIT) of rocks. It can also address the effect of time to obtain isochronous failure surfaces. The effect of scale is also treated, taking into account the size of the element and the loading conditions. A simple continuity parameter, Γ, is used to define the influence of large size defects and extrapolate laboratory tests results to in situ conditions. The use of the $MSDP_u$ criterion is illustrated with a number of experimental results. Application of this criterion to engineered structures was summarized, based on earlier publications and ongoing work.

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REFERENCES

Abdulraheem, A., Roegiers, J.C. & Zamen, M. (1992) *Mechanics of pore collapse and compaction in weak porous rocks*. In: Tillerson, J.R. & Wawersik, W.R. (eds.) *Rock Mechanics*. Rotterdam, Balkema. pp. 233–242.

Akai, K. & Mori, H. (1970) Ein versuch über Bruchmecanismus von Sandstein unter mehrachsigen Spannungszustand. In: *Proceedings of the 2nd International Congress on Rock Mechanics, Belgrade*. Vol. 2, pp. 207–213.

Andreev, G.E. (1995) *Brittle Failure of Rock Materials – Test Results and Constitutive Models*. Rotterdam, Balkema.

Aubertin, M. & Li, L. (2004) A porosity-dependent inelastic criterion for engineering materials. *Int J Plast*, 20(12), 2179–2208.

Aubertin, M. & Simon, R. (1996) *A multiaxial failure criterion that combines two quadric surfaces*. In: Aubertin, M., Hassani, F. & Mitri, H. (eds.) *Rock Mechanics: Tools and Techniques*. Rotterdam, Balkema. pp. 1729–1736.

Aubertin, M. & Simon, R. (1997) A damage initiation criterion for low porosity rocks. *Int J Rock Mech & Min Sci*, 34(3–4), #017, CD-ROM.

Aubertin, M. & Simon, R. (1998) Un critère de rupture multiaxial pour matériaux fragiles. *Can J Civil Engineering*, 25(2), 277–290.

Aubertin, M., Gill, D.E. & Ladanyi, B. (1994) Constitutive equations with internal state variables for the inelastic behavior of soft rocks. *Appl Mech Rev*, 47(6–2), S97–S101.

Aubertin, M., Julien, M.R. & Li, L. (1998) Keynote Lecture: The semi-brittle behavior of low porosity rocks. In: *NARMS'98, Proc 3rd North Am Rock Mech Symp*, Cancun. Rotterdam, Balkema. Vol. 2, pp. 65–90.

Aubertin, M., Li, L., Simon, R. & Khalfi, S. (1999) Formulation and application of a short-term strength criterion for isotropic rocks. *Can Geotech J*, 36(5), 947–960.

Aubertin, M., Li, L. & Simon, R. (2000) A multiaxial criterion for short term and long term strength of rock media. Int J Rock Mech Min *Sci*, 37, 1169–1193.

Aubertin, M., Li, L. & Simon, R. (2001) *Evaluating the large scale strength of rock mass with the* $MSDP_u$ *criterion*. In: Elsworth, D., Tinucci, J.P. & Heasley, K.A. (eds.) *Rock Mechanics in the National Interest: Proceedings of the 38th U.S. Rock Mechanics Symposium, DC Rocks 2001*, Washington DC, 7–10 July 2001. A. A. Balkema. Vol. 2, pp. 1209–1216.

Aubertin, M., Li, L. & Simon, R. (2002) *Effet de l'endommagement sur la stabilité des excavations souterraines en roche dure*. Études et Recherches R-312, Institut de recherche Robert-Sauvé en santé et en sécurité du travail (IRSST), Montreal, Quebec, Canada.

Aubertin, M., Li, L., Simon, R. & Bussière, B. (2003) *A general plasticity and failure criterion for materials of variable porosity.* Technical Report EPM-RT-2003-11, École Polytechnique de Montréal.

Bazant, Z.P. & Chen, E.P. (1997) Scaling of structural failure. *Appl Mech Rev*, 50(10), 593–627.

Bazant, Z.P. & Planas, J. (1998) *Fracture and Size Effect in Concrete and Other Quasi-brittle Materials*. Boca Raton, CRC Press.

Bieniawski, Z.T. (1968) The effect of specimen size on compressive strength of coal. *Int J Rock Mech Min Sci*, 5, 325–335.

Bieniawski, Z.T. (1984) *Rock Mechanics Design in Mining and Tunneling*. Rotterdam, Balkema.

Bieniawski, Z.T. (1989) *Engineering Rock Mass Classification*. New York, John Wiley & Sons.

Brown, E.T. & Yu, H.S. (1988) A model for the ductile yield of porous rock. *Int J Num & Analy Methods Geomech*, 12, 679–688.

Charlez, Ph.A. (1991) Rock Mechanics, Volume 1 – Theoretical Fundamentals. Paris, Editions Technip.

Chen, A.C.T. & Chen, W.F. (1975) Constitutive relations for concrete. *ASCE Journal of the Engineering Mechanics Division*, 101, 465–481.

Coop, M.R. & Willson, S.M. (2003) Behavior of hydrocarbon reservoir sands and sandstones. *J Geotech* Geoenviron *Eng*, 129(11), 1010–1019.

Cristescu, N.D. & Hunsche, U. (1997) *Time Effects in Rock Mechanics*. New York, John Wiley & Sons.

Cunha, A.P. (1993a) *Research on scale effects in the determination of rock mass mechanical properties – The Portuguese experience*. In: Cunha, A.P. (ed.) *Scale Effects in Rock Masses 93*. Rotterdam, Balkema. pp. 285–292.

Cunha, A.P. (1993b) *Scale effects in rock engineering - An overview of the Loen Workshop and other recent papers concerning scale effects*. In: Cunha, A.P. (ed.) *Scale Effects in Rock Masses 93*. Rotterdam, Balkema. pp. 3–14.

Desai, C.S. & Salami, M.R. (1987) Constitutive model for rocks. *ASCE J Geotech Engng*, 113, 407–423.

Desai, C.S. & Siriwardane, H.J. (1984) *Constitutive Laws for Engineering Materials with Emphasis on Geologic Materials*. London, Prentice-Hall.

Drucker, D.C. & Prager, W. (1952) Soil mechanics and plastic analysis on limit design. *Quat Appl Meth*, 10(2), 157–165.

Du, X., Lu, D., Gong, Q. & Zhao, M. (2010) Nonlinear unified strength criterion for concrete under three-dimensional stress states. *J Eng Mech*, 136(1), 51–59.

Elliott, G.M. & Brown, E.T. (1985) Yield of a soft, high porosity rock. *Géotechnique*, 35(4), 413–423.

Franklin, J.A. & Dusseault, M.B. (1989) *Rock Engineering*, New York, McGraw-Hill.

Gerard, C. (1986) Shear failure of rock joints: appropriate constraints for empirical relations. *Int J Rock Mech Min Sci & Geomech Abstr*, 23(6), 421–429.

Germanovitch, L.N., Carter, B.J., Ingraffea, A.R., Dyskin, A.V. & Lee, K.K. (1996) Mechanics of 3-D crack growth under compressive loads. In: Aubertin, M., Hassani, F. & Mitri, H. (eds.) *Rock Mechanics, Tools and Techniques: Proceedings of the 2nd North American Rock Mechanics Symposium*. Balkema, Rotterdam. pp. 1151–1160.

Gerogiannopoulos, N.G. & Brown, E.T. (1978) The critical state concept applied to rock. *Int J Rock Mech Min Sci & Geomech Abstr*, 15(1), 1–10.

Goodman, R.E. (1980) *Introduction to Rock Mechanics*. New York, John Wiley & Sons.

Griffith, A.A. (1921) The phenomena of rupture and flow in solids. *Philos Trans* R *Soc London*, 221A, 163–198.

Griffith, A.A. (1924) The theory of rupture. *Proceedings of the First International Congress Applied Mechanics*. Delft. Partie 1. pp. 55–63.

Hatzor, Y.H. & Palchik, V. (1997) The influence of grain size and porosity on crack initiation stress and critical flaw length in dolomites. *Int J Rock Mech Min Sci*, 34(5), 805–816.

Hjelm, H.E. (1994) Yield surface for grey cast iron under biaxial stress. *ASME J Engng Materials & Technol*, 116, 148–154.

Hoek, E. & Brown, E.T. (1980a) Empirical strength criterion for rock masses. *J Geotech Engng Div*, 106, 1013–1035.

Hoek, E. & Brown, E.T. (1980b) *Underground Excavations in Rock*. London, Institution of Min & Metall.

Hoek, E. & Brown, E.T. (1988) *The Hoek-Brown failure criterion—a 1988 update*. In: *Rock Engineering for Underground Excavations: Proc 15th Can Rock Mech Symp*, Toronto. pp. 31–38.

Hoek, E. & Brown, E.T. (1997) Practical estimates of rock mass strength. *Int J Rock Mech Min Sci*, 34(8), 1165–1186.

Hoskins, E.R. (1969) The failure of thick-walled hollow cylinders of isotropic rock. *Int J Rock Mech Min Sci*, 6, 99–125.

Hudson, J.A. & Harrison, J.P. (1997) *Engineering Rock Mechanics – An Introduction to the Principles*. Oxford, Pergamon Press.

Hunsche, U. (1994) *Uniaxial and triaxial creep and failure tests on rock; experimental technique and interpretation*. In: Cristescu, N.D., & Gioda, G. (eds.) *Visco-plastic Behaviour of Geomaterials, Courses and Lectures*. New York, Springer Verlag, International Center for Mechanical Sciences. pp. 1–51.

Jaeger, J.C. & Cook, N.G.W. (1979) *Fundamentals of Rock Mechanics*. 3rd edition. New York, Chapman and Hall.

Kikiessa-Kisaka, P. 2015. *Évaluation des contraintes en place en profondeur à partir de l'écaillage des trous de forage*. M.Sc.A. thesis, École Polytechnique de Montréal.

Ladanyi, B. & Archambault, G. (1970) *Simulation of shear behavior of a jointed rock mass*. In: *Proc 11th US Symp on Rock Mech*, Berkeley. pp. 105–125.

Lade, P.V. (1993) Rock strength criteria - The theories and evidence. In: Hudson, J.A. (ed.) *Comprehensive Rock Engineering – Principles, Practice and Projects*. Oxford, Pergamon Press. Vol. 1, pp. 255–284.

Lade, P.V. (1982) Three parameter failure criterion for concrete. *ASCE J Eng Mech*, 108, 850–863.

Lama, R.D. (1974) *The uniaxial compressive strength of jointed rock mass*. Prof. L. Muller Festschrift, Univ Karlsruhe. pp. 67–77.

Lau, J.S.O. & Gorski, B. (1991) *The Post-Failure Behaviour of the Lac du Bonnet Grey Granite*. CANMET: Divisional Report MRL 91-079 (TR).

Lemaitre, J.A. (1992) *Course on Damage Mechanics*. New York, Springer-Verlag.

Li, C. & Nordlund, E. (1993) Deformation of brittle rocks under compression – with particular reference to microcracks. *Mech Mater*, 15, 223–239.
Li, L. & Aubertin, M. (1999) Estimation des contraintes dans les roches à partir de l'écaillage autour de trous de forage. *Rev Franç Geotech*, 89, 3–11.
Li, L. & Aubertin, M. (2000) Un critère de rupture multiaxial pour les roches avec une anisotropie planaire. Proceedings, 52nd Canadian Geotechnical Conference, 15–18 October 2000, Montreal, Quebec. CGS. Vol. 1, pp. 357–364.
Li, L. & Aubertin, M. (2003) A general relationship between porosity and uniaxial strength of engineering materials. *Can. J. Civ. Eng*, 30(4), 644–658.
Li, L. & Aubertin, M. (2009) An elastoplastic evaluation of the stress state around cylindrical openings based on a closed multiaxial yield surface. *Int. J. Numer. Anal. Methods Geomech*, 33(2), 193–213.
Li, L., Aubertin, M. & Simon, R. (1999) Stability analyses of underground openings using a multiaxial failure criterion. In: Fernandez, G. & Bauer, R.A. (eds.) Geo-Engineering for Underground Facilities: *Proceeding of the 3rd National Conf Geo-Institute ASCE*, Urbana-Champaign. pp. 471–482.
Li, L., Aubertin, M. & Simon, R. (2000) Analyses de la stabilité d'un tunnel non supporté avec un critère de rupture multiaxial. *J Can Tunnelling*, 2000, 119–130.
Li, L., Aubertin, M. & Simon, R. (2001) *Stability analyses of underground openings using a multiaxial failure criterion with scale effects*. In: Wang, S., Fu, B. & Li, Z. (eds.) *Frontiers of Rock Mechanics and Sustainable Development in the 21st Century: Proceedings of the 2nd Asian Rock Mechanics Symposium*, 11–14 September 2001, Beijing, China. Lisse, A.A. Balkema. pp. 251–256.
Li, L., Aubertin, M., Simon, R. & Bussière, B. (2005) Formulation and application of a general inelastic locus for geomaterials with variable porosity. *Can Geotech J*, 42(2), 601–623.
Li, L., Aubertin, M, Simon, R., Deng, D. & Labrie, D. (2007) *Chapter 97, Influence of scale on the uniaxial compressive strength of brittle rock*. *Rock Mechanics: Meeting Society's Challenges and Demands*. London, Taylor & Francis Group. pp. 785–79.
Li, L., Aubertin, M. & Shirazi, A. (2010) Implementation and application of a new elasto-plastic model based on a multiaxial criterion to assess the stress state near underground openings. *ASCE Int J Geomech*, 10(1), 13–21.
Lu, D., Du, X., Wang, G., Zhou, A. & Li, A. (2016). A three-dimensional elastoplastic constitutive model for concrete. *Comput Struct*, 163, 41–55.
Lubliner, J. (1990) *Plasticity Theory*. New York, McMillan Publishing.
Lundborg, N. (1974) *A statistical theory of the polyaxial strength of materials*. In: Advances in Rock Mechanics: Reports of Current Research; Proceedings of the Third Congress of the International Society for Rock Mechanics, Themes 1-2, Denver, Colorado, September 1–7, 1974. National Academy of Sciences, Washington, DC. Vol. II, Part A.
Martin, C.D. & Chandler, N.A. (1994) The progressive fracture of Lac du Bonnet granite. *Int J Rock Mech Min Sci & Geomech Abstr*, 31, 643–659.
Maso, J.C. & Lereau, J. (1980) Mechanical behavior of Darney sandstone in biaxial compression. *Int J Rock Mech Min Sci & Geomech Abstr*, 17, 109–115.
Mazanti, B.B. & Sowers, G.F. (1965) Laboratory testing of rock strength. In: Proc Symp Testing Techniques for Rock Mech, Seattle. pp. 207–227.
McClintock, F.A. & Walsh, J.B. (1962) Friction on Griffith cracks under pressure. In: Proceeding of the 4th US National Congr Appl Mech, Berkeley. Vol. 2, pp. 1015–1021.
Medhurst, T.P. & Brown, E.T. (1998) A study of the mechanical behaviour of coal for pillar design. *Int J Rock Mech Min Sci*, 35(8), 1087–1105.
Meredith, P.G. (1990) *Fracture and failure of brittle polycrystals: an overview*. In: Barber, D.J. & Meredith, P.G. (eds.) Deformation Processes in Minerals, Ceramics and Rocks. London, Unwyn Hyman. pp. 5–47.

Michelis, P. (1987) True triaxial yielding and hardening of rock. *J Geotech Engng*, 113(6), 616–635.

Mills, L.L. & Zimmerman, R.M. (1970) Compressive strength of plain concrete under multiaxial loading conditions. ACI Proc - J Am Concrete Inst, 67(10), 802–807.

Mitri, H.S., Edrissi, R. & Henning, J. (1994) *Finite element modelling of cable-bolted stopes in hard rock underground mines*. Reprint No. 94-116, Society for Mining, Metallurgy and Exploration Inc.

Nguyen, D. (1972) *Un concept de rupture unifié pour les matériaux rocheux denses et poreux*. Ph.D. thesis, École Polytechnique de Montréal – Université de Montreal.

Nova, R. (1986) An extended Cam-Clay model for soft anisotropic rocks. *Comput Geotech*, 2, 69–88.

Ottosen, N.S. (1977) A failure criterion for concrete. *ASCE J Eng Mech*, 103, 527–535.

Papanikolaou, V.K. & Kappos, A.J. (2007) Confinement-sensitive plasticity constitutive model for concrete in triaxial compression. Int J *Solids Struct*, 44(21), 7021–7048.

Paterson, M.S. (1978) *Experimental Rock Deformation – The Brittle Field*. Wien, Springer-Verlag.

Patton, F.D. (1966) Multiple modes of shear failure in rock joints in theory and practice. Proceedings of the 1st Congress of the International Society of Rock Mechanics, Lisbon. Vol. 1, pp. 521–529.

Pellegrino, A. (1970) Mechanical behaviour of soft rock under high stresses. In: Proceedings of the 2nd International Conference on Rock Mechanics, Beograd. Vol. 2, pp. 173–180.

Pratt, H.R., Black, A.D., Brown, W.S. & Brace, W.F. (1972) The effect of specimen size on the mechanical properties of unjointed diorite. *Int J Rock Mech Min Sci*, 9, 513–529.

Ramamurthy, T. & Arora, V.K. (1994) Strength predictions for jointed rocks in confined and unconfined states. *Int J Rock Mech Min Sci & Geomech Abstr*, 31(1), 9–22.

Schmidtke, R.H. & Lajtai, E.Z. (1985) The long term strength of Lac du Bonnet granite. *Int J Rock Mech Min Sci & Geomech Abstr*, 22, 461–465.

Shao, J.F. & Henry, J.P. (1991) Development of an elastoplastic model for porous rock. *Int J Plasticity*, 7(1), 1–13.

Shao, J.F., Khazraei, R. & Henry, J.P. (1996) *Application of continuum damage theory to borehole failure modelling in brittle rock*. In: Aubertin, M., Hassani, F. & Mitri, H. (eds.) *Rock Mechanics: Tools and Techniques*. Rotterdam, Balkema. pp. 1721–1728.

Sheorey, P.R. (1997) *Empirical Rock Failure Criteria*. Rotterdam, Balkema.

Singh, J., Ramamurthy, T. & Rao, G.V. (1989) *Strength of rocks at great depth*. In: Maury, V. & Fourmaintraux, D. (eds.) *Rock at Great Depth*. Rotterdam, Balkema. pp. 37–44.

Skrzypek, J.J. & Hetnarski, R.P. (1993) *Plasticity and Creep—Theory, Examples, and Problems*. Boca Raton, CRC Press.

Theocaris, P.S. (1995) Failure criteria for isotropic bodies revisited. *Engineering Fracture Mechanics*, 51(2), 239–264.

Thorel, L. (1994) *Plasticité et Endommagement des Roches Ductiles - Application au Sel Gemme*. Ph.D. thesis, École des Ponts et Chaussés.

Wastiels, J. (1979) *Failure criteria for concrete under multiaxial stress states*. Proceedings of the Colloquium on Plasticity in Reinforced Concrete, International Association for Bridge and Structural Engineering, Lyngby. pp. 3–10.

Wong, R.H.C., Chau, K.T. & Wang, P. (1996) Microcracking and grain size effect in Yuen Long marbles. *Int J Rock Mech Min Sci & Geomech Abstr*, 33(5), 479–485.

Wong, T.F., Szeto, H. & Zhang, J. (1992) Effect of loading path and porosity on the failure mode of porous rocks. *Appl Mech Rev*, 45(8), 281–293.

You, M.Q. (2015) Strength criterion for rocks under compressive-tensile stresses and its application. *J Rock Mech Geotech Eng*, 7, 434–439.

Yu, M.H. (2002) Advances in strength theories for materials under complex stress state in the 20th Century. *Appl Mech Rev*, 55(3), 169–218.