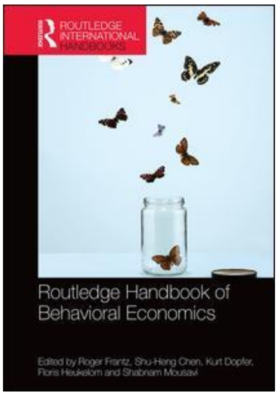

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ECONOMIC BEHAVIOUR AND AGENT-BASED MODELLING

Matthias Mueller and Andreas Pyka

Introduction

Since the 1980s, continuous improvements in computer technologies have remarkably changed scientists' possibilities to develop and apply computer simulations. Simulations as a scientific tool offer new ways to explore the dynamics of complex models in various disciplines. They enable us to process scientists' thought models into artificial test laboratories in which we can systematically analyse the models' outcomes.

Within the broad field of simulation techniques, the so-called agent-based modelling (ABM) approach has gained increasing momentum, not only for economics but also in many other scientific disciplines. The ABM approach takes the perspective of the system building elements and focuses on the actions and interactions of these entities as the relevant actors within the system. This ABM perspective is accompanied by the attempt to represent actors of economic systems in a more realistic fashion, thereby overcoming the shortcomings of approaches limited to representative agents which by definition ignore heterogeneity and the related implications of interacting heterogeneous agents. In this vein, the research objectives of ABM show a strong similarity with behavioural economics, where deviations from the assumed theoretical behaviour play an outstanding role.

The value of this computational modelling approach becomes clear when we look at the experience gained during the 2010 financial crisis. As Jean-Claude Trichet phrased it:

> When the crisis came, the serious limitations of existing economic and financial models immediately became apparent. [...] Macro models failed to predict the crisis and seemed incapable of explaining what was happening to the economy in a convincing manner. [...] We need to deal better with heterogeneity across agents and the interaction among those heterogeneous agents. [...] Agent-based modelling dispenses with the optimization assumption and allows for more complex interactions between agents. Such approaches are worthy of our attention.
>
> *(Trichet, 2010)*

One reason for the dissatisfaction with the currently dominant paradigm in economics is substantiated in the compulsory assumption of "rational expectations" (Farmer & Foley, 2009). Empirical evidence shows that this assumption is clearly at odds with real human behaviour (see for example the experiments conducted by Güth et al. (1982) and Berg et al. (1995)). To overcome this constrictive paradigm, we clearly need new insight into human behaviour and the development of institutions, which are prominently addressed by the behavioural economics approach. Given the enormous degree of complexity, however, we also need a methodological framework, such as ABM, to incorporate the individual and heterogeneous behaviour of economic actors into models that are capable of portraying the emerging dynamics.

In an early attempt Bob Axelrod characterizes ABM by "the existence of many agents who interact with each other with little or no central direction" (Axelrod, 1997). A final consensus of what ABM is and how scientists can use it to increase our understanding of the complex economic system so far is not existent. The aim of this chapter is to shed some light on the ABM approach as a promising and possibly necessary tool in the scientist's toolbox. We focus, therefore, on the most central methodological issues, reviewing the current state of literature and highlighting the possibilities of this still somewhat disputed approach and emphasizing the complementary relationship of ABM and behavioural economics.

We start in the second section with an introduction to the three pillars of ABM: *modelling, agents and simulation*, which are characteristic for all agent-based models. In the third section we then introduce important methodological aspects that every agent-based modeller must be aware of. Finally, we conclude with a number of remarks and a brief outline of the complementary relationship between ABM and behavioural economics.

The three pillars of ABM

Despite the awareness that ABM has gained for many years, a common protocol for this method is still missing. Driven by the increasing computer resources, different scientific disciplines discovered the versatile possibilities offered by this modelling approach, which lead to different notions and understandings of ABM used in economic science.

Figure 28.1 Flock of birds created by the BOIDS algorithm

To illustrate the diversity of notions and emphasis, ABM also is also labelled as: *ABM* (Epstein & Axtell, 1996), *agent-based simulation modelling* (Polhill et al., 2001), *multi-agent simulation* (Ferber, 1999; Gilbert & Troitzsch, 2005), *multi-agent-based simulation* (Edmonds, 2001), *agent-based social simulation* (Doran, 2001; Downing et al., 2001), *individual-based configuration modelling* (Judson, 1994), *multi-agent systems* (Bousquet & Le Page, 2004), and *agent-based computational economics* (Tesfatsion, 2002).

Despite missing consent concerning the appropriate notion for this scientific method, the understanding of ABM is very similar (see also Hare & Deadman (2004) for a discussion on this issue). In the following section, we will describe three pillars of agent-based modelling: *modelling, agents and simulation*, which will frame our understanding of ABM.

Modelling from an agent-based perspective

The most important element of ABM is its *bottom-up perspective*—describing a system from the perspective of its constituent units, i.e. the agents (Bonabeau, 2002). In short, building the model from the bottom up means, letting complex macroscopic systems emerge from the interactions of microscopic entities (Epstein & Axtell, 1996; Axelrod, 1997). A good example illustrating this underlying principle is the artificial life program BOIDS by Reynolds (1987), which reproduces the complex behaviour of a flock of birds (see Figure 28.1). Instead of treating a flock as a self-contained unit, Reynolds was able to recreate the behaviour of a flock of birds by disaggregating the flock into birds and, therefore, building the model from the bottom up using only three simple behavioural rules for the birds:

1. Separation—avoid crowding neighbours (short range repulsion);
2. Alignment—steer towards average heading of neighbours; and,
3. Cohesion—steer towards average position of neighbours (long range attraction) (Reynolds, 1987).

Although this example could be considered trivial, it shows that the behaviour of complex systems can only be reproduced adequately by taking into account the individual behaviour, that is, the actions and interactions of the system building units. This also illustrates how ABM differs from another numerical method that enjoys a certain popularity in economics, namely system dynamics. On the system level, the rich patterns of possible dynamics remain hidden, whereas it is in the explicit focus of ABM.

In economics, the ABM approach follows the same idea as Reynolds proposed, modelling (macroscopic), such as economic systems through the actions and interactions of (micro-)entities, such as firms, and universities and so on. ABM hereby departs from the top-down perspective of mainstream economic models. Instead of representative individuals constrained by strong consistency requirements associated with equilibrium and an Olympian rationality, ABM describes *heterogeneous* entities living in complex systems that evolve through time (Windrum et al., 2007).

A definition of agents

Although the bottom-up perspective of ABM is without doubt a key characteristic, there are other approaches following the same logic of reasoning. In contrast, for example to the related family of Cellular Automates (Wolfram, 1986) or Microsimulations (Orcutt et al., 1986), the ABM approach centres on a representation of agents in a more realistic way. Focusing on the individual behaviour of economic actors, agent-based models can display important concepts such as for example *heterogeneity* and *bounded rational behaviour* of agents, which leads to all sorts of variation in their modelled behaviour.

An agent represents a dynamic entity, which can be assigned by the modeller with an individual role exhibiting a variety of characteristics. Although a final consensus has not been reached, in the literature it is often claimed that agents may possess the following properties (Wooldridge & Jennings, 1995):

Autonomy: Agents are autonomous entities with little or no central direction and have control of their actions.
Social ability: Agents can interact with other agents (e.g. receiving or sending information about locations or other internal states of others).
Reactivity: Agents have a perception of their environment (e.g. the landscape they are in).
Pro-activeness: Agents exhibit goal-directed behaviour, taking the initiative.

The list of possible characteristics of agents can be extended in several ways. In their original paper, Wooldridge and Jennings (1995) had already named more human characteristics (e.g., knowledge, belief, intention, obligation and emotions) as possible additional features of agents (Wooldridge & Jennings, 1995; Jennings et al., 1998).

In ABM approaches economic actors are what they are, that is, autonomous and heterogeneous entities embedded in an environment that is created by the actions and interactions of these agents (Gilbert & Troitzsch, 2005). In contrast to traditional modelling approaches, the variety of agents and their behaviour are not restricted to fit into an analytical framework. Depending on the problem under investigation and the scope of the corresponding model, agents can flexibly represent any kind of economic actor. On an aggregated level, this can be firms, universities, governmental bodies, and so on; on an individual level, agents can be employees, scientists, consumers, households and so on. However, from an abstract point of view, any independent component of a system can basically be considered to be an agent (Bonabeau, 2002; Macal & North, 2005).

It is important to emphasize that with ABM we can specifically relax unrealistic assumptions about the agents and their behaviour. While in most mainstream models strong assumptions are required in order to guarantee in principle an analytical solvability (Farmer & Foley, 2009), in ABM every agent is endowed with an individual set of initial states, which allows for the representation of characteristic features and a representation of individual behaviour. Consequently, the ABM approach is able to incorporate the manifold insights from behavioural economics about human and institutional behaviour into fruitful models.

In particular, agents within an agent-based model can be assumed to have only limited information about the environment and the behaviour of other agents, limited foresight about the scope of decisions or other resource limitations, such as memory, and so on (Edmonds, 1999). Building on that, agent-based models are capable of displaying true heterogeneity of agents (Macal & North, 2005). Heterogeneity in this sense means that agents are modelled as individual entities with individual states, and with individual behaviours. Heterogeneity in the model can be assigned by the modeller according to the requirements of the problem under investigation, that is, agents may be endowed with different levels of resources, initial knowledge stocks, strategies, reference systems etc. Additionally, heterogeneity is endogenously created within the model through the actions and interactions of agents themselves.

Second, as the behaviour of agent-based models is not restricted to obtain an analytical solution, agents may be assumed to behave as rationally bounded entities (Pyka & Fagiolo, 2007; see also Das (2006) for a detailed discussion). Since the ABM approach focuses directly on the individual, it allows for an intentional non-rational design of economic decisions which, for example, allows for experimental adaption and learning. This enables us to model the effects of psychological principles as for example *reference dependence*, *loss aversion* and *non-linear probability weighting* postulated by the famous prospect theory by Kahneman and Tversky (1979).

The complexity of a model with individual and heterogeneous agents, however, quickly reaches a level where for example the traditional analytical framework fails to offer any solution. A way-out for this problem can be found in computational simulation environments where even complex models can be studied in detail.

Simulation as in-silicio *laboratories*

As a third pillar in our understanding of the ABM approach, we have to consider that agent-based models often are implemented within a simulation environment. Seeing simulation as a form of quasi-experiments, in principle, simple agent-based models can be carried out without the help of simulation tools. A prominent example for this is the famous *Segregation model* by Schelling (1969), which was originally conducted on a chessboard using coins of different colours.

The complexity of any model, however, grows exponentially with the magnitude of the model's assumptions, quickly reaching a level where computational support is necessary, such as in the processing of experimental results. Especially through the steady improvements in computer performance, but also in the progress made on the software side (e.g. object-oriented languages and simulation environments especially dedicated to ABM, such as NetLogo, LSD—Laboratory for Simulation Development, Repast—Recursive Porous Agent Simulation Toolkit etc.), today's simulations act as laboratories where agent-based models can be created and studied *in-silicio* (Pyka & Fagiolo, 2007).

For ABM simulation, tools facilitate a detailed look into the complex interplay between the model's assumptions and the resulting outcome. By building an agent-based model within a computer simulation environment, we have a tool at hand, which helps us to systematically observe and analyse the complex dynamics created by the actions and interactions of agents both on a macro as well as on a micro level. With a computational simulation, we are able to observe all relevant information of the simulation as it progresses. In contrast to real world experiments, simulations offer the possibility of recreating and repeating experiments with the same initial conditions. This gives us the opportunity to systematically alter model parameters and assumptions, and hence leads us to a comprehensive understanding of the model's outcome.

The complexity involved, however, still limits the possible scope of ABM. Although today's computer performance allows for models with an unforeseen range, the complexity of models will always be limited by the capabilities to process the data obtained by the simulation, especially on the researchers' side.

Using ABM as a scientific tool

Despite the new and promising perspective ABM offers, it is also necessary to deal with the question of how ABM can contribute to the scientific endeavour. If used properly, ABM offers researchers a new perspective on complex interplay within economic systems.

Managing the complexity

Although ABM at its core makes a huge step towards a more realistic model of economic systems, one cannot expect a fully detailed picture. As with any model, an agent-based model is designed as a purposeful representation of a system rather than an exact and precise attempt to display real systems (Starfield, 1990). "Purposeful" in the broadest possible sense can be understood as a model that helps the scientist to answer questions that are of interest (Minsky, 1965).

Managing the complexity within a model, that is, finding the right level of complexity, is one of the key challenges for any agent-based modeller. As previously stated, the possibility to implement agent-based models within a simulation environment, the increasing performance of computer systems, and constantly improving methods for data analysis and visualization enable researchers to create models of unforeseen complexity and detail.

To start, almost by definition, we will be hardly able to define something as the optimum level of complexity that should be strived for by ABM in general. Yet there is an extreme that we need to be aware of: if the complexity of the model is reaching a level where we are no longer able to understand the processes involved, then the experiments conducted are of little interest and we cannot understand these artificial complex systems any better than we understand the real ones (Gilbert & Terna, 2000; Axtell & Epstein, 1994).

The level of complexity of an ABM is determined by the actors that the modeller aims to include and the number of assumptions we consider relevant for the model. In particular, for models of economic systems there is a broad range of possible actors and assumptions that can be relevant. The modeller must decide carefully to what extent the elements of the model are necessary or negligible, facing a common trade-off: while models aiming at prediction need descriptive accuracy, models designed for explanatory power should be rather simple (Axelrod, 1997).

Considering the right strategy for building the complexity within ABM, the debate has triggered a rich methodological discussion in which we find three distinct modelling strategies. First, following the *KISS* (*Keep It Simple, Stupid*) strategy one should start with a simple model, which may be extended if necessary. A special case of the KISS strategy is the so called TAPAS approach ("Take A Previous model and Add Something"). Here one starts with an existing model and successively complicates it with incremental additions (Pyka & Fagiolo, 2007).

In contrast, the *KIDS* (*Keep-It-Descriptive, Stupid*) strategy follows the idea of starting with a descriptive model first, which is then, if possible, simplified (Edmonds & Moss, 2005). An illustrative example for the KISS modelling strategy is Schelling's model of segregation in North American cities (Schelling, 1969). In this model, Schelling uses a simple grid for the representation of a city in order to model neighbourhood relationships. He succeeds in identifying the mechanisms that lead to strong clustering patterns of ethnic groups, even if this was only mildly intended in the individual behaviour of agents (Schelling, 1969).

A good example for the KIDS strategy is the model of water demand by Edmonds and Moss (2005), which includes an extremely rich set of varying behaviour rules, preference systems, water consuming devices (power showers, water-saving washing machines etc.), pricing systems, and policy options. With the help of this rich set of elements, backed by empirical observations, the authors intend to model water demand in a region close to the real water demand.

It is important to understand that, in general, the KISS and the KIDS strategies do not differ concerning the degree of complexity. In principle, it is rather a question of how to get there, although in reality it is probably inevitable that the choice for a strategy concerning building the relevant assumptions will end in models with considerably different levels of complexity.

Without going into too much detail, the debate has many facets and a set of additional strategies must be included for a full picture (Pyka & Fagiolo, 2007). It is, however, important to note, that agent-based research should not be oppressed by the fear of complexity. Any claim that ABM is limited to investigate only simple dynamics in small systems is neglecting the possibilities of ABM. Applying ABM for the study of economic systems, however, requires a detailed methodological understanding of this research method and additional inputs from other disciplines; that is, through a profound understanding of realistic behavioural heuristics.

Two ways of using agent-based models

As stated before, the bottom-up perspective of ABM builds on the actions and interactions between heterogeneous agents to analyse the multilevel effects on the overall system, the environment and the agents themselves. Through this, ABM gives a unique understanding of the processes of the interplay between micro and macro levels of a complex system.

The particular role of ABM for the scientific endeavour still is under debate. Facing critiques questioning the scientific value of ABM the modeller needs to be aware of how ABM can be used to deepen our understanding. Based on the literature we can distinguish between two (as will be argued later) complementary perspectives facing different questions.

On the one hand, following the idea introduced by Epstein and Axtell (1996), ABM can be used in a *generative* perspective. ABM in this sense focuses on the possibilities to display the emergence of complex macroscopic system behaviour by the actions and interactions of agents on a micro level. In other words, agent-based models can be designed to find micro-specifications that can explain (from a generative perspective *grow*) a macro level phenomenon of interest (Epstein, 1999). In the literature, this is often put on a level of reproducing stylized facts through ABM to validate models.

To name just a few examples for interesting models reproducing stylized facts, Thomas Schelling (1969) showed, as already mentioned, with his famous Segregation model, that the macroscopic pattern of segregation in cities can be explained by even a minor preferences of inhabitants for neighbourhoods with the same colour. In 1996, Epstein and Axtell reproduced, among other things, right-skewed wealth distributions based on their seminal Sugarscape model. The scope of the model has been extended during the past years continuously, implementing new features carrying forward new aspects to the model (see Epstein (1999) for a list of other interesting models reproducing stylized facts). The logic of reasoning behind the generative perspective of modelling stylized facts constitutes also a severe methodological caveat of the ABM approach which is stressed also by Epstein (1999) or Gilbert and Terna (2000). In principle, it is possible that similar macroscopic system behaviour might be generated by models which refer to different assumptions with respect to the behavioural rules of agents on the micro level. The agent-based modeller simply cannot presume that the microscopic behaviour of agents found to reproduce stylized facts is in fact an accurate and relevant description of the phenomena of interest. For this reason, a final proof for the validity of an agent-based model which reproduces stylized facts is hard to give.

On the other hand, using agent-based models is not limited to reproducing stylized facts and thereby finding *possible* explanations for emerging macroscopic patterns. While the starting point for agent-based models following the generative idea is patterns on the macro level, another strand of models in the literature is based on the possibility to perform a wide range of numerical experiments and, hence, analyse what emerging patterns arise based on micro-specifications of the model. The focus here is on the improvement of our understanding of the dynamic processes within a complex system. ABM from this perspective acts as a laboratory for computational experiments created *in-silicio* (Pyka & Fagiolo, 2007; Leombruni, 2002). As Axelrod puts it:

> Simulation is a third way of doing science. Like deduction, it starts with a set of explicit assumptions. But unlike deduction, it does not prove theorems. Instead, a simulation generates data that can be analysed inductively. Unlike typical induction, however, the

> simulated data comes from a rigorously specified set of rules rather than direct measurement of the real world.
>
> *(Axelrod, 1997)*

Detached from the need to reproduce stylized facts, ABM can also be seen as a scientific method to increase our understanding of complex systems. While the starting point of ABM in a generative sense is macroscopic patterns, the starting point of ABM as an *in-silicio* laboratory is a set of specifications for the micro level; that is, a well-grounded set of behavioural rules on the agent level. Tesfatsion (2006) distinguishes here between two different objectives of ABM. First, ABM can be used to get a *normative understanding*, evaluating whether designs proposed for economic policies, institutions, and processes will result in socially desirable system performance over time. Second, ABM allows for *qualitative insights and theory generation*. The objective here is to understand economic systems through a systematic examination of their potential dynamical behaviours under alternatively specified initial conditions. Using ABM as an *in-silicio* laboratory, however, comes at a cost. To gain wide acceptance, the model need to be validated through a broad set of empirical micro level data.

The need for verification, validation and calibration

The most common critique against agent-based models is perceived lack of robustness. To start, any modeller needs to be aware of the possibility of simple failures in the implementation of the model in a computer simulation. This *verification* of the computer program is a crucial step for the credibility of a model.

Verification simply asks whether the model does what we think it is supposed to do (Ormerod & Rosewell, 2009). Verification, however, is more than just looking for bugs in the computer code. For example, Dawid and Kopel (1998) note: "we have to be aware of the fact that simulation results may crucially depend on implementation details which have hardly any economic meaning". In other words, even though the technical code might be correct, the way the model is translated and implemented in the computer software might lead to biased results which have strong effects on the model's outcome, yet were unintended by the modeller. Especially for complex models, it is therefore necessary to make the implementation of the agent-based model transparent for other researchers. This can be done either by providing the original simulation code or by making main procedures public, including the pseudo code in the publication.

Despite the correct implementation of the simulation model, the modeller is also often confronted with critiques questioning whether the model is an accurate representation of the real world from the perspective of the model's intended applications, that is, the model's validity (Ormerod & Rosewell, 2009). Following the distinction made in the third section, we can differentiate between two ways of *validation*: that is, the *input validation* and *output validation*.

While the latter refers to the matching of model results against acquired real world data, the former regards ensuring that the fundamental structural, behavioural and institutional conditions incorporated in the model reproduce the main aspects of the actual system (Bianchi et al., 2008).

Without going into much detail, this issue is still under great debate and several strategies have been developed to approach the problem of validity of agent-based models, such as the *indirect calibration* approach and the *Werker–Brenner* approach. The underlying concept of these validation strategies is an elaborate multilevel approach where input and output validation are combined. Starting with a model designed to reproduce stylized facts, the modeller can use the micro-specifications found to be valid to replicate the macro patterns as the starting point for a wide set of simulation experiments aiming to give further insights into the dynamical behaviours of the model (see, for example, Windrum et al. (2007), Ormerod & Rosewell (2009), and Werker & Brenner (2004) for interesting discussions on this issue).

Although, without doubt, the validation of an agent-based model is of particular interest, especially if we want to derive valid outcomes such as policy recommendations, we have to be aware that the complexity of ABM always invites criticism. Despite any thorough validation of the model, we may be confronted with questions whether all elements of the model are necessary or if other, so far missing, elements are relevant too and, hence, should be included. Second, the possibility of performing validation is restricted by the set of relevant data available. For validation of agent-based models, we need more than some widely accepted macroeconomic stylized facts. As ABM builds on the actions and interactions of heterogeneous economic actors, this approach requires a fundamentally new understanding of the behaviour of these actors.

Conclusions

Shifting the focus from an oversimplifying perception of economic systems to a more realistic one, ABM is designed to overcome the limiting possibilities of a traditional analytical framework. Central to this new approach is its exceptional perspective of economic actors, treating economic agents as heterogeneous and individual actors that build economic systems from the bottom up.

Using this method, however, comes at a cost. We need to be aware of how ABM can be used to deepen our understanding of complex economic processes. Although ABM at its core makes a huge step towards a more realistic model of economic systems, one cannot expect a fully detailed picture. As with any model, an agent-based model is designed as a purposeful representation of a system. Purposeful in the sense that ABM can be used in a *generative* way, reproducing macro level patterns and hence finding *possible* explanations on the micro level, but also as an *in-silicio* test laboratory where we study the outcome of different micro level specifications.

Especially for the latter case, despite any effort to ensure validity of our models, the complexity created within ABM invites critiques. To counter them, we need a better understanding of the behaviour of economic actors but also well accepted standard models. So far, ABM cannot aim for including all possible aspects of an economy. In contrast, we need at this point to focus on a new joint understanding of basic economic processes, emerging from a profound acknowledgement of human behaviour. Building on that, we can stepwise increase the complexity of our models, gaining descriptive accuracy and hence increasing the predictive power of ABM.

In this light, it is also important to emphasize the complementary relationship of ABM with behavioural economics, offering the possibility for fruitful cross-fertilization. In behavioural economics, instead of a general optimization approach, decision rules are frequently informed by psychological insights of human behaviour, which serve as a heuristic description of decision processes. Empirical observations confirm in many instances a deviation from a theoretically derived optimal solution. As these deviations are systematic and not random, their explanation is of high scientific interest. In many cases, these empirical observations can be explained by behavioural economists with the application of decision heuristics for comparatively simple and artificial cases. ABM now allows for an extension of these comparatively simple and artificial cases towards models with a higher degree of complexity. With this, we can test whether unexpected feedbacks and phase transitions resulting from the interaction of heterogeneous agents might have a strong impact on the expected results. In contrast, ABM finds a rich collection of decision heuristics in the behavioural economics literature that can be used to program agents' decision rules, framing a new and profound understanding of economic systems.

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